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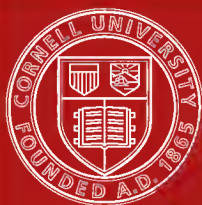
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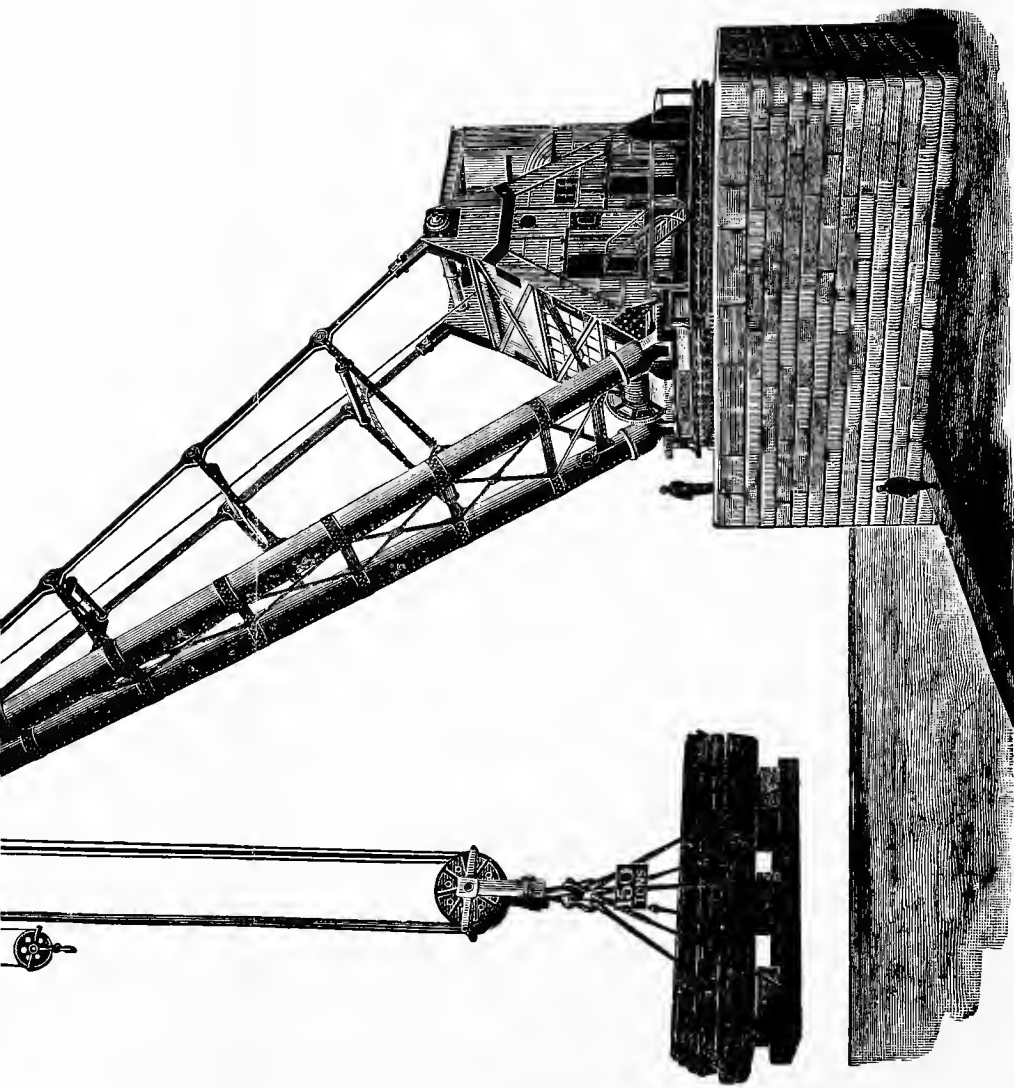
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of Education, and Honours Certificates of the City
and Guilds of London Institute, in Mechanical
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BY

ANDREW JAMIESON, M.INST.C.E.,

FORMERLY PROFESSOR OF ENGINEERING IN THE GLASGOW AND WEST OF SCOTLAND
TECHNICAL COLLEGE; MEMBER OF THE INSTITUTION OF ELECTRICAL ENGINEERS;
FELLOW OF THE ROYAL SOCIETY, EDINBURGH; AUTHOR OF TEXT-BOOKS
ON STEAM AND STEAM ENGINES, APPLIED MECHANICS AND
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P R E F A C E

TO THE SEVENTH EDITION.

It has been found necessary to still further subdivide this wide and all-important subject of Advanced Applied Mechanics and Mechanical Engineering.

In order to do so with the least departure and derangement of the previous volumes and editions, it has been advisable and convenient to follow the recent subdivision of this subject as stated in the "Rules and Syllabus of Examinations applying to the Election of Associate Members of The Institution of Civil Engineers."

Moreover, this particular method of subdivision is practised by several Universities and Technical Colleges. It is also being advocated by Teachers in connection with the Boards of Education, and, to a certain extent, by those connected with the City and Guilds of London Examinations in Mechanical Engineering.

Consequently, Volume I. will deal with "Applied Mechanics" proper, Volume II. will discuss and give practical illustrations of "Strength and Elasticity of Materials," Volume III. will be confined to "The Theory of Structures," Volume IV. to "Hydraulics, Hydraulic and Refrigerating Machinery," whilst Volume V. will be greatly enlarged, and treat upon "The Theory of Machines."

Separate Contents and Index have been carefully arranged for each Volume. These enable students to find the details and pages where the different subjects are treated. The Author's system of Engineering Symbols, Abbreviations, and Index Letters have been printed at the beginning of each volume.

It is thus hoped, that the size and cost of each volume will suit the requirements of every Student of Engineering.

The Author has again to thank his chief assistant, Mr John Ramsay, A.M.Inst.C.E., for his help, and his Publishers for the special care with which they have prepared these five volumes. The Author is specially indebted to Professor Alexander MacLay, B.Sc., C.E., and the Technical Publishing Company, Ltd., Manchester, for permission to abstract certain parts and figures from his excellent book on "*Locs* in Mechanical Drawing, including Point Paths in Mechanisms." These additions appear in Vol. V. upon the "Theory of Machines," along with the new Lectures dealing with the "Kinematics of Machinery."

The Author will feel much obliged to Engineers, Teachers, his B.Sc. and C.E. "Correspondence Students," also to any other Students of Engineering, for hints which may tend to further enhance the attractiveness and usefulness of these works.

ANDREW JAMIESON.

Consulting Engineer and Electrician.

16 ROSSLYN TERRACE, KELVINSIDE,
GLASGOW, September, 1909.

INSTRUCTIONS FOR ANSWERING HOME EXERCISES.

(As used in my *Engineering and Electrical Science Correspondence System*.)

1. Use ordinary foolscap paper, and write on the left side *only*, leaving the facing page blank for my corrections and remarks.

2. Put the date of the Exercises at the left-hand top corner; your Name and Address in full, the name of the Subject or Section, as well as number of Lecture or Exercise, in the centre of the first page. The number of each page should be put in the right-hand top corner.

3. Leave a margin $1\frac{1}{2}$ inches wide on the left-hand side of each page, and in this margin place the *number* of the question *and nothing more*. Also, leave a clear space of *at least* 2 inches deep between your answers.

4. Be sure you understand *exactly* what the question requires you to answer, then give *all* it requires, but *no more*. If unable to fully answer any question, write down your own best attempt and state your difficulties.

5. Make your answers concise, clear, and exact, and always accompany them, if possible, by an *illustrative sketch*. Try to give (1) Side View, (2) Plan, (3) End View. Where asked, or advisable, give Sections, or Half Outside Views and Half-Sections for (1), (2), and (3).

6. Make all sketches large, open, and in the centre of the page. Do not crowd any writing about them. Simply print sizes and index letters (or names of parts), with a bold Sub-heading of what each figure or set of figures represent.

7. The character of the sketches will be carefully considered in awarding marks to the several answers. Neat sketches and "*index letters*," having the first letter of the name of the part, will always receive more marks than a bare written description.

8. All students are strongly recommended to *first* thoroughly study the Lecture, *second* to work out each answer in scroll, and *third* to compare it with the question and the text-book in order to see that each item has been answered. The final copy should be done without *any aid or reference* to the scroll copy or to *any text-book*. This acts as the best preparation for sitting at Examinations.

9. Reasonable and easily intelligible contractions (*e.g.*, mathematical, mechanical or electrical, and chemical symbols) are permitted.

10. Each corrected answer which has the symbols R.W. marked there-upon must be carefully re-worked as *one* of the set of answers for the following week.

ANDREW JAMIESON.

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- VOL. V.**—Theory of Machines.—Tooth, Friction, Belt, Rope, Chain and Miscellaneous Gearing, with their Applications to Machines.—Shapes and Strengths of Teeth.—Automatic Tooth-Cutting Machines.—Velocity-Ratio and Power Transmitted by Gearing.—Motion and Energy.—Practical Applications to Governors, Flywheels, and Centrifugal Machines.

MECHANICAL ENGINEERING SYMBOLS, ABBREVIATIONS, AND INDEX LETTERS

USED IN VOLUMES I. TO V.

OF PROFESSOR JAMIESON'S "APPLIED MECHANICS."

Prefatory Note.—It is very tantalising, as well as a great inconvenience to Students and Engineers, to find so many different symbol letters and terms used for denoting one and the same thing by various writers on mechanics. It is a pity, that British Civil and Mechanical Engineers have not as yet *standardised* their symbols and nomenclature as Chemists and Electrical Engineers have done. The Committee on Notation of the Chamber of Delegates to the International Electrical Congress, which met at Chicago in 1893, recommended a set of "Symbols for Physical Quantities and Abbreviations for Units," which have ever since been (almost) universally adopted throughout the world by Electricians.* This at once enables the results of certain new or corroborative investigations and formulæ, which may have been made and printed anywhere, to be clearly understood anywhere else, without having to specially interpret the precise meaning of each symbol letter.

In the following list of symbols, abbreviations and index letters, the *first* letter of the chief noun or most important word has been used to indicate the same. Where it appeared necessary, the *first* letter or letters of the adjectival substantive or qualifying words have been added, either as a following or as a subscript or suffix letter or letters. For certain specific quantities, ratios, coefficients and angles, small Greek letters have been used, and I have added to this list the complete Greek alphabet, since it may be refreshing to the memory of some to again see and read the names of these letters, which were no doubt quite familiar to them when at school.

* These "Symbols for Physical Quantities and Abbreviations for Units" will be found printed *in full* in the form of a table at the commencement of Munro and Jamieson's *Pocket-Book of Electrical Rules and Tables*. If a similar recommendation were authorised by a committee composed of delegates from the chief Engineering Institutions, it would be gladly adopted by "The Profession" in the same way that the present work of "The Engineering Standards Committee" is being accepted.

TABLE OF MECHANICAL ENGINEERING QUANTITIES, SYMBOLS, UNITS
AND THEIR ABBREVIATIONS.

(As used in Vols. I. to V. of Prof. Jamieson's "Applied Mechanics.")

Quantities.	Symbols.	Defining Equations.	Practical Units.	Abbreviations of the Practical Units.
FUNDAMENTAL.				
Length, . . .	L, l	...	{ Yard, . . . yd. Foot, . . . ft. Inch, . . . in.	
Mass, . . .	M, m	...	{ Pound, . . . lb. Second, . . . s.	
Time, . . .	T, t	...	{ Minute, . . . m. Hour, . . . h.	
GEOMETRIC.				
Surface, . . .	S, s	$S = L^2$	{ Square foot, . . sq. ft. Square inch, . . sq. in.	
Volume, . . .	V	$V = L^3$	{ Cubic foot, . . cb. ft. Cubic inch, . . cb. in.	
Angle, $\angle$. .	$\left\{ \begin{array}{l} \alpha, \beta \\ \theta, \phi \end{array} \right\}$	$\alpha = \frac{\text{arc}}{\text{radius}}$	{ Degree, . . . 1° Minute, . . . $1'$ Second, . . . $1''$ Radian = $\frac{180^\circ}{\pi}$. rn.	
MECHANICAL.				
Velocity, . . .	v	$v = \frac{L}{T}$	Foot per second, .	$\frac{\text{ft.}}{\text{s.}}$
Angular velocity, .	ω	$\omega = \frac{v}{L} = \frac{\theta}{t}$	{ Revs. per second, . Revs. per minute, . Radians per second, .	{ r.p.s. r.p.m. $\frac{\omega}{s}$
Acceleration, . .	a, g	$a = \frac{v}{T}$	Foot per sec. per sec.	$\frac{\text{ft.}}{\text{s}^2}$
Force, . . .	F, f	...	{ Pound weight (gravitational unit),	{ lb. wt. (or lb.)
	W, w	$F = Ma$	{ Poundal (absolute unit),	{ pdl.
Pressure (per unit area),	p	$p = \frac{F}{s}$	Pound per sq. inch,	lb. $\square''$
Work, . . .	(Wh)	$Wh = FL$	Foot-pound, . .	ft.-lb.
Potential energy, .	E_p	$E_p = Wh$	Foot-pound, . .	ft.-lb.
Kinetic energy, . .	E_k	$E_k = \frac{Wv^2}{2g}$	Foot-pound, . .	ft.-lb.
Power or activity, .	HP	$\text{H.P.} = \frac{Wh}{T}$	{ Horse power, . . H.P. Ft.-lb. per min., .	{ ft.-lb./m.
Moment of inertia, .	I	$I = Mk^2$	{ Ft.-lb. per sec., ...	{ ft.-lb./s. lb.-ft. ²
Density, . . .	ρ	$\rho = \frac{M}{V}$	{ Pound per cb. ft., .	{ $\frac{\text{lb.}}{\text{ft.}^3}$
			{ Pound per cb. in., .	{ $\frac{\text{lb.}}{\text{in.}^3}$

OTHER SYMBOLS AND ABBREVIATIONS IN VOLS. I. TO V.

A for Areas.	x, y, z for Unknown quantities.
B, b „ Breadths.	Z „ Modulus of section.
C, c, k „ Constants, ratios.	Z_t „ „ tension.
c.g. „ Centre of gravity.	Z_c „ „ compression.
D, d „ Diameters depths, deflections.	
D_1, D_2, D_3 „ Drivers in gearing.	Δ, δ, d for Differential signs which are prefixed to another letter; then the two together represent a very small quantity.
E „ Modulus of elasticity.	e, e „ Represents base of Napierian Logs = 2.7182; for example, $\log_e 3 = 1.1$.
e „ Velocity ratio in wheel gearing.	η „ Efficiency.
F_1, F_2, F_3 „ Followers in gearing.	λ „ Length ratio of ship to model.
f, f_t „ Forces of shear and tension.	μ „ Coefficient of friction.
H, h „ Heights, heads.	π „ Circumference of a circle $\div$ its diameter.
H.P., h.p. „ Horse-power.	ρ „ Radius of curvature, radian.
B.H.P. „ Brake horse-power.	
E.H.P. „ Effective „	Σ for Symbol for sum total of a number of quantities.
I.H.P. „ Indicated „	$\int_a^x$ „ Sign of integration or summation between limits 0 and x .
k „ $\left\{ \begin{array}{l} \text{Radius of gyration, or,} \\ \text{Coef. of discharge in hydraulics.} \end{array} \right.$	$\sim$ „ Sign for the difference between two quantities.
N, n „ Numbers—e.g., number of revs. per min., number of teeth, &c.	$\square$ „ Sign for square—e.g., $10 \square'' = 10$ square inches.
P, Q „ Push or pull forces.	— „ Sign over two letters, $\overline{PQ}$, for a force acting from P to $\rightarrow$ Q, means that they represent a vector quantity, which has (1) magnitude, (2) direction, (3) sense.
$R_1 R_2$ „ Reactions, resultants, radii, resistances.	$\supseteq$ „ Sign for equal to or greater than.
s „ $\left\{ \begin{array}{l} \text{Seconds, space, surface.} \\ \text{Displacement, distance.} \end{array} \right.$	$\leq$ „ Sign for equal to or less than.
S F „ Shearing force.	
T M „ Torsional moment.	
T R „ Torsional resistance.	
B M „ Bending moment.	
M R „ Moment of resistance.	
R M „ Resisting moment.	
T_a, T_s „ Tensions on driving and slack sides of belts or ropes, &c.	
W_L, W_T, W_U „ Lost, total, and useful work.	

GREEK ALPHABET.

A	α	Alpha.	I	ι	Iota.	P	ρ	Rho.
B	β	Beta.	K	κ	Kappa.	Σ	σ or ς	Sigma.
Γ	γ	Gamma.	Λ	λ	Lambda.	T	τ	Tau.
Δ	δ	Delta.	M	μ	Mu	Υ	υ	Upsilon.
E	ϵ	Epsilon.	N	ν	Nu	Φ	ϕ	Phi.
Z	ζ	Zeta.	Ξ	ξ	Xi.	X	χ	Chi.
H	η	Eta.	O	o	Omicron.	Ψ	ψ	Psi.
Θ	θ	Theta.	Π	π	Pi.	Ω	ω	Oméga.

VOLUME III.

ON

THEORY OF STRUCTURES AND GRAPHIC STATICS, WITH APPLICATIONS TO ROOFS, CRANES, BEAMS, GIRDERS, AND BRIDGES.

LECTURE I.

CONTENTS. —Graphic Statics—A Framed Structure—Classification of Frames—Firm Frames—Deficient Frames—Redundant Frames—Conditions of Equilibrium—Bow's Method of Lettering—Solution of a Triangular Frame—Reciprocal Figure for a Joint—Definition of a Strut—Definition of a Tie—Stress Diagram—Determination of the Kind of Stress in a Bar—Firm Quadrilateral Frame—Firm Triangular Frame—Firm Frame—Firm Frame with Mansard Outline—Questions.

Graphic Statics is the Science and Art of determining by scale drawings the total stresses in the various parts of a structure. The forces transmitted through each part of a structure may be ascertained either by calculation or by graphical construction. The former method is extremely tedious, except in very simple cases, whereas the latter is not only rapid, but also affords a self-evident means of checking the accuracy of the solution.

DEFINITION.—A Framed Structure consists of an assemblage of rigid bars, so arranged, that the stresses in them are principally push or pull and by the use of which, external forces may be transmitted or modified.

A structure is different from a machine in so far as, the former transmits force while the latter transmits energy. This means that the parts of a structure are assumed to be at rest while those of a machine must be in motion.

In this section we assume, unless otherwise stated :—

(1) That the point of crossing of two or more bars is a frictionless joint, and that the external forces act *only* on the joints of the frame.

(2) That all the members, bars, or links are able to withstand either push or pull, and are consequently termed "rigid bars."

(3) That each bar is incapable of being appreciably deformed under the action of the stress it may have to carry.

For the complete specification of a force, we must know the following four elements:—

- (1) The point or place of application.
- (2) The line of action—*i.e.*, the line along which the force is acting.
- (3) The direction or way the force acts along its line of action.
- (4) The magnitude—*i.e.*, the number of units of force.

Classification of Frames.—(1) **Firm Frames** are those which have *just* sufficient bars to prevent change of shape, and any bar may therefore be lengthened or shortened without stressing any of the other members.

(2) **Deficient Frames** are those which have *not* sufficient bars to prevent deformation, and the joints must therefore be made stiff in order to resist change of form.

(3) **Redundant Frames** are those which have *more* bars than are necessary to resist distortion. In frames of this kind we cannot alter the length of certain bars without stressing one or more of the other members. Further, the frame may be self stressed if the redundant bars be badly fitted, and the stresses in the various members are indeterminate unless their yield-*ingness* be taken into account.

Conditions of Equilibrium.—There must be no translation. This is assured if the diagram of external forces is a closed polygon. In other words, their "*Vector Sum*" must = 0.*

There must be no rotation. This is satisfied if:—

- (1) The external forces have no resultant movement round any point that may be chosen.
- (2) The line of action of the resultant of all except two of the forces passes through the point of intersection of the lines of action of these two forces.

If a number of external forces act upon a structure and keep it at rest, and, if we have to determine graphically the relations among these external forces, we must know at least:—

Either.—All the elements of all the forces except one and nothing about that one.

Or.—All the elements of all the forces except two, and about one of these two its line of action. About the other, one point in its line of action.

In the former case, we determine the resultant of all the given external forces by any method, and the last or balancing force (that is, the one we know nothing about) has (1) its point of application anywhere in the line of action of the resultant, (2) its line of action coincident with the line of action of the resultant, (3) its direction or way opposite to that of the resultant, and (4) its magnitude is the same as that of the resultant.

The second case will be clear by a reference to Fig. 1.

BA is the resultant of the external forces, 1.2.3...(n - 2), acting on the body or frame. DC is the line of action of the

* See the note at the end of this Lecture for a definition of Scalar, Vector, and Rotor.—A.J.

$(n - 1)^{\text{th}}$ force, and E the point chosen as a point in the line of action of the n^{th} force.

If three forces act upon a body and keep it at rest their lines of action must all pass through one point. Thereby, the line of action FE, of the n^{th} force may be determined, since it must pass through O the point of intersection of BA with DC. Then by an application of the triangle of forces the magnitudes and ways or directions of the $(n - 1)^{\text{th}}$ and n^{th} forces may be determined.

Bow's Method of Lettering.—In Fig. 2 we have an example of Bow's method of lettering a system of forces. It will be seen

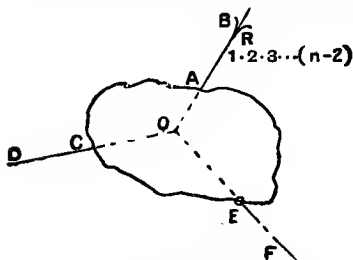


Fig. 1.—RELATION AMONG EXTERNAL FORCES.

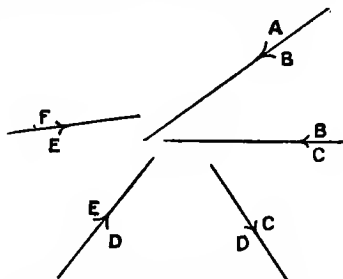


Fig. 2.—ILLUSTRATION OF BOW'S METHOD OF LETTERING.

that every force has one letter on each side of its line of action. This is in order to name the force. Thus, we speak of the forces AB, BC, CD, DE, and EF. Again, each letter has been used twice, excepting A and F. This would indicate that one force was wanting or required to be determined:—viz., the force AF. This force may be the resultant or the equilibrant as the case may be; or, if on drawing the polygon of forces, F coincides with A (that is, the magnitude of FA is zero), then the system is in translationary equilibrium.

In Fig. 3, we have the forces acting at the joints of the triangular frame XYZ, named by Bow's method.

The forces which keep in equilibrium the joints X, Y, and Z, are:—

For the joint X—

The force BC (all the elements of which are known).

The action of the stress * CD; and,

The action of the stress DB.

* As is usual, in treatises on this subject, the word *stress*, throughout Vol. III., means the *total force* transmitted by the bar, and not the force per unit of cross area.

For the joint Z—

The action of the stress B D ;
 The action of the stress D A ; and,
 The supporting force A B.

And for the joint Y—

The action of the stress A D ;
 The action of the stress D C ; and,
 The supporting force C A.

The supporting force C A, has been represented by a curved dotted line to indicate that all we know about it is, its point of application.

The point X might be called the joint B C D ; the point Y the joint C A D ; and the point Z the joint A B D, since the letters naming a joint have been used to name the forces acting at that joint.

In Fig. 3, we have used the letters A, B, and C each four times and the letter D six times. In practice, this is avoided by lettering, as indicated in Fig. 4. Then the forces and bars will have the same names as before. Success in graphic solu-

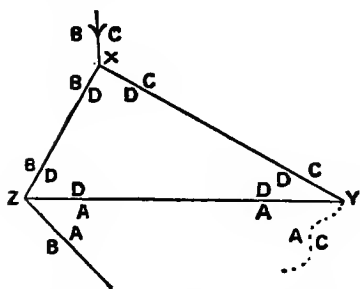


Fig. 3.—BOW'S METHOD OF LETTERING A TRIANGULAR FRAME.

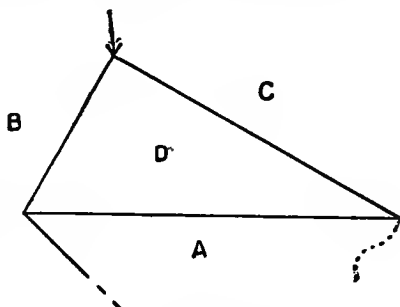


Fig. 4.—BOW'S METHOD IN PRACTICE.

tions depends in a great measure on correct lettering, and on correct assumptions having been made, first, with regard to the total number of external forces that act on the frame, and second, with regard to what is known about the various elements of these external forces.

One great advantage of the Graphic Method of Solution is, that our attention is always being directed to the correctness of any assumptions that have been made. If the Stress Diagram

closes, we may safely consider the solution to be correct for the assumptions made; but, if it does not, some assumption is wrong or something has been left out.

Correct lettering is accomplished when every external force and every bar has one letter and only one on each side of it.

All the external forces must be applied at the joints of the frame, but if any should act at a point other than the end of a bar, then two equivalent parallel forces must be applied to the member under consideration, one at each end. By equivalent parallel forces is meant two forces which, applied as stated, would have the given force as their resultant.

The lines of action of the external forces must not fall inside the frame, but must be drawn outside, as in Fig. 4.

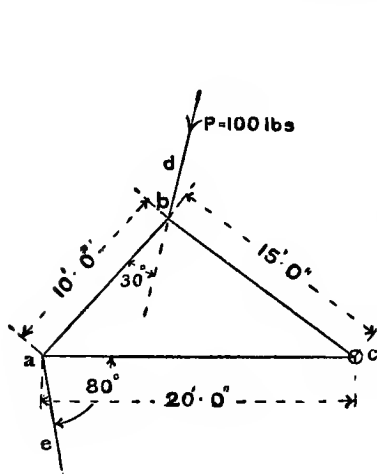


Fig. 5.—SKETCH OF FRAME.

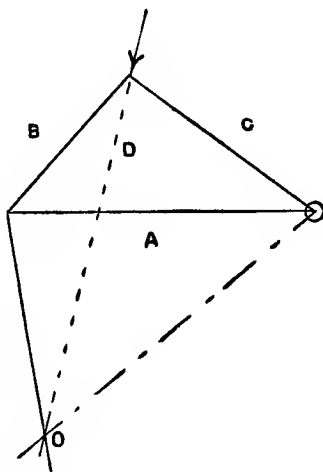


Fig. 6.—FRAME DIAGRAM.

Solution of a Triangular Frame.—*Given*, the triangular frame abc , and the force P , completely specified as follows, viz. :—

Its point of application, b ;

Its line of action, db ;

Its way or direction from d towards b ; and,

Its magnitude, P lbs. Also,

The line of action of one of the supporting forces, ae .

And finally, a point c , in the line of action of the other supporting force.

It is required to find the remaining elements of the supporting forces, also the magnitudes and kind of stresses in the bars.

We begin by drawing the Frame Diagram (Fig. 6) to scale. This scale should be as large as possible, say, in this case, $\frac{1}{4}$ inch representing 1 foot. Then letter the Frame Diagram by Bow's method. Now, let the lines of action of the forces $A B$ and $B C$, on being produced meet in O . Then *for no rotation*, the line of action of the other supporting force $C A$ (as indicated by the chain dotted line) must pass through O , and also through the joint $D C A$, as given. Thus the line of action of the force $C A$ is determined.

For no translation, the triangle of forces is applied, and will give the magnitudes and ways of the supporting forces, as indicated by Fig. 7. The scale used should be as large as convenient, say 1 inch representing 40 lbs.

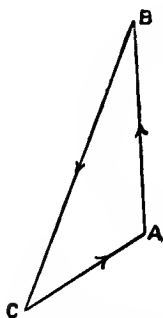


Fig. 7.—DIAGRAM FOR
EXTERNAL FORCES.

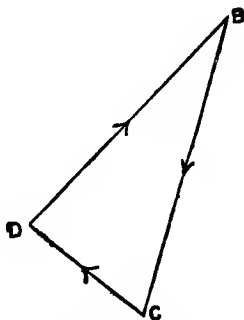


Fig. 8.—RECIPROCAL FIGURE
FOR JOINT B C D

DEFINITION.—If from a point, a number of lines radiate, and if a polygon be drawn which has its sides either all parallel to, or all at right angles to corresponding radiating lines, then this Polygon is called the Reciprocal of the Point.

Thus, the triangle $B C A$, Fig. 7, may be called the reciprocal of the point $B A C$ or O , in Fig. 6.

We can now draw the reciprocal figure for any one of the joints of the triangular frame. Because, we know all about the external forces acting at each of these joints; and further, not more than two bars meet at each joint.

If more than two bars meet at a joint, then we must know, in addition to all the external forces acting at that joint, the stresses in all the bars except two, before the reciprocal can be drawn.

Fig. 8 is the reciprocal figure for the joint B C D. It is drawn to the same scale and in exactly the same manner as Fig. 7, viz.:—B C parallel and equal to the external force B C; C D parallel to the bar C D; and D B parallel to the bar D B.

The length of the lines C D and B D, in Fig. 8 (measured to the same scale as that used for B C) determine the magnitudes of two forces which, acting in conjunction with the external force B C, would keep the joint B C D, at rest. The two forces C D and D B, are the actions on the joint of the stresses in the bars C D and D B, and therefore measure the magnitudes of the stresses in these bars. The arrow-heads give the ways or directions along the line of centres of the bars of the actions C D and D B.

Figs. 9 and 10 are drawn to the same scale and in the same way as Fig. 8, and represent the reciprocals for the joints B D A and A D C respectively.

From Figs. 9 and 10 we get similar information regarding the bars A D and D B, and their actions on the joint D B A, and the bars C D and D A, and their actions on the joint A D C, to that derived from Fig. 8 regarding the joint B C D.

In the reciprocal figure for the joint B C D, Fig. 8, the way

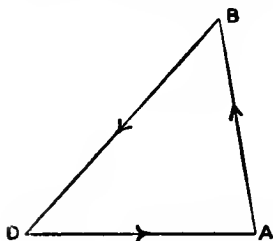


Fig. 9.—RECIPROCAL FOR JOINT B D A.

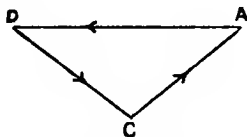


Fig. 10.—RECIPROCAL FOR JOINT A D C.

of the action on the joint B C D, of the stress in the bar C D, is towards the left and upwards, while in Fig. 10 the way of the action of the stress in the same bar on the joint A D C, is towards the right and downwards.

We will now explain the cause of this apparent contradiction in the two reciprocals. The reciprocal for the joint B C D, shows that the way of the action on the joint B C D, of the stress in the bar D C, is towards the pin B C D—that is, pushing it. (This is indicated in Fig. 11 by the small arrow.) Then from Newton's third law the pin B C D, must push the bar with an equal and opposite force.

If the bar DC pushes the pin BCD, it must also push the pin CAD. For this reason, that no bar can simultaneously push a pin at one end of itself and pull one at its other end. This is what the reciprocal for the joint ADC, indicates.

DEFINITION OF A STRUT.—When the reciprocal, for a joint indicates that the way of the action of the stress in a bar is towards the joint, then that bar is under compression and is called a strut.

On reference to the reciprocals for the joints BDA and ADC, it will be seen that the bar DA, is pulling at the pins BDA and ADC. But, by the action and reaction law, the pins will pull at the ends of the bar, and this means that the bar DA, is under tensional stress of an amount measured by the length of the line DA, in the reciprocal figures.

DEFINITION OF A TIE.—When the reciprocal for a joint indicates that the way of the action of the stress in a bar is away from the joint, then that bar is under tension, and is called a tie.

In Fig. 12, the reciprocals for the three joints of the frame and the one for the point O, have been combined into one diagram, which may be called either the Stress Diagram or the reciprocal of the Frame Diagram.

DEFINITION.—Two figures are reciprocal when every point in the one has a corresponding reciprocal in the other.

For example, the point C, in Fig. 12 has the lines DC, AC, and BC, meeting in it. If we refer to the Frame Diagram, Fig. 6, we find that, the bar CD, the force BC, and the force AC, form the reciprocal for this point C in Fig. 12.

We do not put arrow heads on the Stress Diagrams; they would lead to confusion, and are quite unneces-

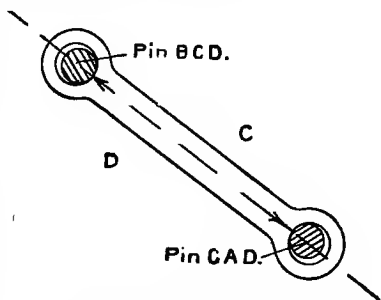


Fig. 11.—THE BAR CD.

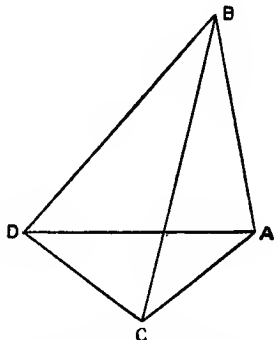


Fig. 12.—COMBINATION OF THE RECIPROCAL, OR STRESS DIAGRAM.

sary. Take, for example, the line A D, Fig. 12, from the reciprocal of the joint A B D, we would require an arrow head pointing from D towards A; while, from the reciprocal of the joint A D C, an arrow head would require to point the other way. (The double arrow on the line A D, points to the fact that a stress has no way.)

It will have been observed :—

(1) That when Figs. 7, 8, and 9 have been drawn they give all the information that was intended to be derived from drawing Fig. 10—viz., Fig. 8 gave the magnitude of O D, and Fig. 9 that of D A.

(2) That when we place the reciprocal for the joint A B D, on the reciprocal for the point O, as in Fig. 12, we have only to join D to O in order to complete the diagram.

These two observations point out that we have too much information; the excess is due to the finding of the point O. This frame is one of a class where we may find the stresses without first finding all the elements of the reactions or supporting forces.

Stress Diagram.—We shall now show how to determine the Stress Diagram direct from the Frame Diagram—i.e., without first finding the reciprocals for the joints, and combining them into one.

It is quite immaterial as to which way we go round a structure—i.e. (referring to Fig. 6), whether we go from A to B and then to C, or the other way round. We shall find it to be an advantage to go round every structure in the same way as the hands of a watch. By doing so we shall find that the Stress Diagram will always lie to the left hand of the external force polygon. This will enable us to know where to begin the external force polygon in order to leave room for the Stress Diagram.

Referring to Fig. 6, where we are not supposed to know either the point O, or the line of action of the force C A, let us plot out therefrom the Stress Diagram, Fig. 12.

(1) Draw B C, in Fig. 12, parallel to the line of action of the external force B C in the Frame Diagram, Fig. 6, and containing 100 units, to some convenient scale, say 1 inch to represent 40 lbs. The correct lettering of this line is a very important part of the work. Since we are going round the frame in the direction of the hands of a watch—that is, from B to C—then B must be put at the top end and C at the bottom end of the line just drawn so as to indicate the way of the force correctly. If this point is attended to, little trouble will be experienced in drawing the diagrams.

(2) Draw from the last point found (viz., C) a line parallel to some force or bar which has C as one of the letters for its name; for example C D.

(3) Draw from the other end B of the line BC, a line parallel to the bar BD, and mark the point of crossing of the two lines D.

(4) Through D, the last point determined, draw DA parallel to the bar DA, and from some of the other points found draw a line parallel to a force whose line of action is known. Now, CA cannot be used because we only know its point of application, but we know the line of action of BA. Then drawing from B, in Fig. 12, a line parallel to the line of action of the supporting force AB, we determine the point A.

(5) On joining C with A we obtain a line parallel to the line of action of the supporting force CA, and the length of this line, CA, measures the magnitude of the force.

(6) Measuring the lines in Fig. 12 with the scale used to draw down the line BC, we obtain the magnitudes of the stresses in all the bars and of the two supporting forces.

How to Determine the kind of Stress in a Bar.—We will begin with the consideration of the forces which act on the left-hand joint—viz., the joint BDA, in Fig. 12.

Success in this part of the work depends almost entirely upon giving to each bar meeting in the joint under consideration its proper name—i.e., by letters in their proper order.

Since we have gone round the external forces in drawing the Stress Diagram from B to C, etc.—that is, in the direction of the hands of a watch—we must go round each joint of the structure in the same way when naming the bars meeting in that joint.

The bars meeting in the joint BDA, would therefore be called, the bar BD, the bar DA, and the supporting force or reaction AB. Having thus determined the name of the bar, we then refer to the Stress Diagram in order to find the way in which the stress in that bar acts with regard to the joint.

Take for example the horizontal member in the Frame Diagram, Fig. 13 (this member is called the tie rod or tie beam, since it ties the lower ends of the rafters together), its name with reference to the joint BDA, is DA. Now, in the Stress Diagram D is on the left of A, and, therefore, the stress in the bar DA, acts from left to right (i.e., from D to A) with respect to the pin at the joint BDA. This means that the bar DA, is pulling at the pin BDA, and therefore the pin BDA, pulls at the bar, thereby putting the bar into tension.

Similarly the stress in the bar BD (called a rafter) acts, so far as the joint BDA is concerned, in the direction indicated by BD in the Stress Diagram—that is, from B to D. The bar BD, is therefore pushing at the joint BDA, and is thus put into compression by the reaction of the pin BDA.

RULE TO DETERMINE THE KIND OF STRESS IN A BAR.—Take

the letters on each side of the Bar in the Frame Diagram, in the same order with respect to the joint on which the Bar acts, as we have taken the letters on each side of the External Forces acting on the Frame. Then along the line in the Stress Diagram, which is named after the Bar, from the first letter

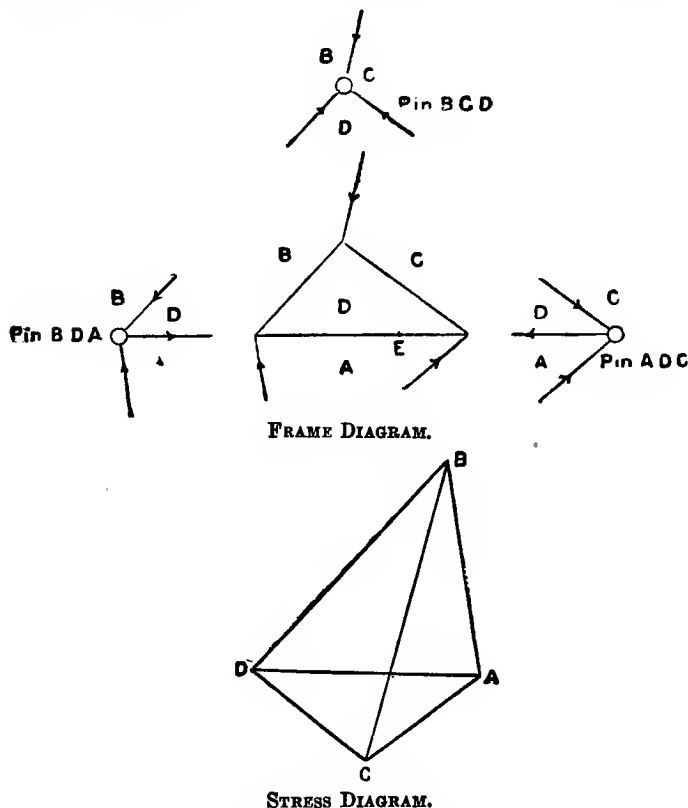


Fig. 13.—STRESS ACTION ON PINS.

towards the second, gives the way of the stress' action with respect to the joint under consideration. If the way is towards the joint the stress in the Bar is Compression or Push, and the Bar is called a Strut. If the way is away from the joint the stress is a Tension or Pull, and the Bar is called a Tie.

The above rule may also be applied to a point in a bar. Take for example, the point E, in the tie rod, and suppose we want to find how the left-hand portion of the tie rod acts upon the section at E. Then the name of the left-hand portion of the tie rod with respect to E is A D, and from the Stress Diagram this acts from right to left—that is, away from E—and is therefore pulling at the section.

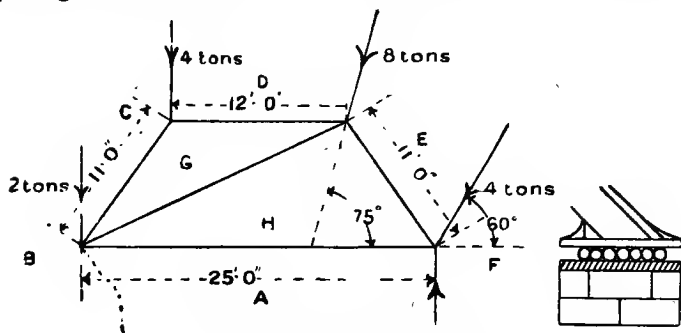


FIG. 14a.—FRAME DIAGRAM.

FRAME SOLVABLE WITHOUT KNOWING ALL ABOUT REACTIONS.

Notice that the tie rod, with respect to the left-hand joint, is called D A, and with respect to the right-hand joint would be called A D, and similarly with any other bar in the frame.

The above rule for the kind of stress does away with the use of arrow-heads and of supplementary diagrams.

The action of all the bars on the pins of the frame are shown in the small diagrams surrounding the Frame Diagram of Fig. 13.

Firm Quadrilateral Frame.—This frame is one of a type which allows a solution to be found without having first determined all the elements of the reactions.

We shall assume that the right-hand end rests on rollers, as indicated in Fig. 14a. Consequently the line of action of the reaction is practically vertical. If it simply slides instead of rolling, then the reaction is inclined to the normal at an angle equal to the angle of friction, and inclined

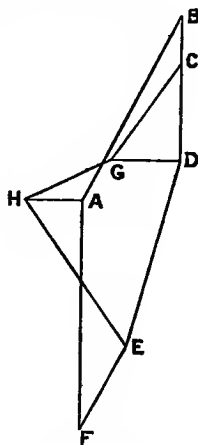


Fig. 14b.—STRESS DIAGRAM.

to that side of the normal which will oppose the motion of the frame. The left-hand end of the frame in Fig. 14a is assumed to be anchored to the wall by bolts, &c. All we know about the left-hand reaction is its point of application.

If we suppose both ends of a frame to be anchored, then we only know the points of application of the reactions, and we assume that their lines of action are parallel to each other and to the line of action of the resultant of the external forces.

We begin by drawing the Frame Diagram to as large a scale as possible, and then indicate the external forces at the joints by their lines of action. The right-hand reaction is indicated by a vertical line, and the left-hand reaction by a dotted curved line, as shown in Fig. 14a. We then letter the diagram according to Bow's method.

In drawing the Stress Diagrams, we shall always go round the Frame Diagrams clockways.

We begin by drawing a line parallel to the line of action of the first force or load BC. This line should contain as many units of length as BC contains units of force, which in this example is 2 tons.*

Then draw CD parallel to the line of action of the load CD, and containing 4 units of length corresponding to the 4 tons load. Next draw DE parallel to the line of action of the load DE, and EF parallel to the line of action of the load EF, representing 8 units and 4 units, respectively.

The line BCDEF is called the Line of Loads.

In order to complete the Stress Diagram we shall begin with the joint CDG, which is the only joint of which we have sufficient data. Draw from the point C in the "Line of Loads" a line parallel to the bar CG, and from D a line parallel to the bar DG. The intersection of these two lines is called the point G. From G draw GH parallel to the bar GH, and from E draw EH parallel to the bar EH. This determines the point H. Then draw HA parallel to the bar HA, and from F draw a line parallel to the line of action of the reaction FA. The intersection of these two lines fixes the point A. Joining A with B gives the finishing line of the Stress Diagram. The line AB in the Stress Diagram is parallel to the line of action of the left-hand reaction.

By applying the rule for the kind of stress, we can determine from the diagram all we may wish to know—*e.g.*, with respect to the top right-hand joint, the diagonal member is called HG.

* The scale for the diagram should be as large as convenient. A rough guess may be made by adding all the loads together, and assuming that this will be the total vertical length of the diagram.

On reference to the Stress Diagram we see that the way of its action is from H to G which means pushing at the joint. Therefore, the diagonal member is in compression, and so on for the other members.

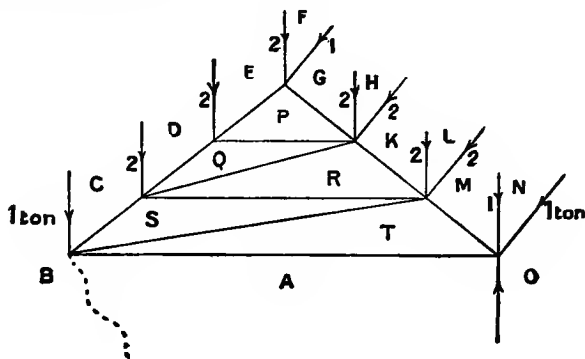


FIG. 15a.—FRAME DIAGRAM.
FRAME WITH WIND PRESSURE.

The magnitudes of the stresses are measured by the lengths of the lines in the Stress Diagram.

The polygon BCDEFA is called the polygon of external forces.

Firm Triangular Frame.

—This frame, Fig. 15a, can also be solved without knowing all about the reactions.

The right-hand end of the frame is assumed to be resting on rollers, while the left-hand end is anchored to the wall. The vertical loads on the Frame Diagram represent the action of gravity on the roofing, such as slates, &c., which is assumed to be uniformly distributed over the surface.

In Fig. 15a, the rafters are shown divided into three equal

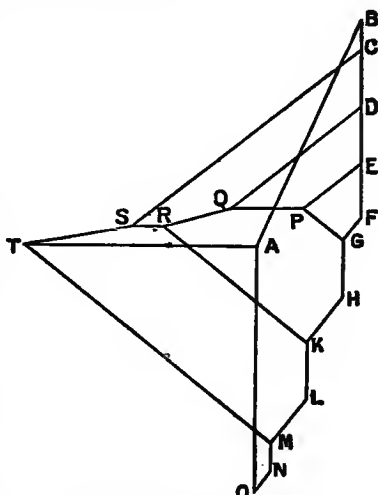


FIG. 15b.—STRESS DIAGRAM.

parts called bays; and, since the joint at each end of a bay must carry one half of the uniformly distributed load over that bay,

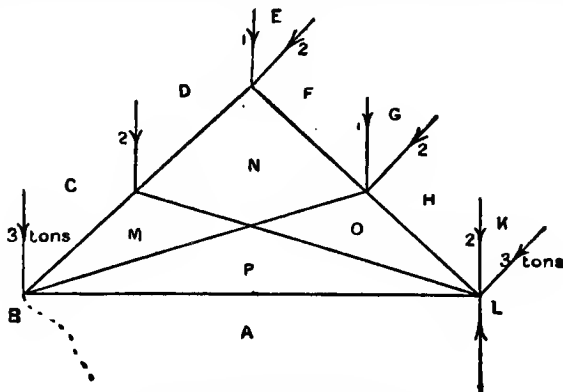


FIG. 16a.—FRAME DIAGRAM.
FIRM FRAME WITH A QUADRILATERAL PART.

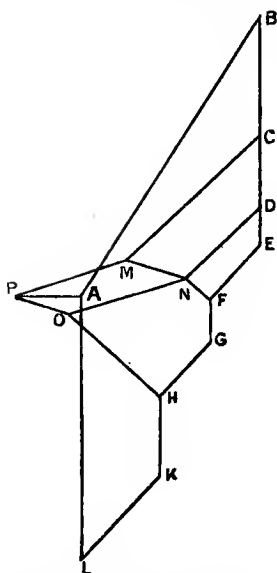


FIG. 16b.—STRESS DIAGRAM.

the vertical loads will have the proportions shown by the numbers on the Frame Diagram. Wind pressure is also assumed to be uniformly distributed, and is reckoned as so many lbs. per square foot normal to the rafters. This is indicated on the right-hand side of the Frame Diagram.

Note.—When wind pressure acts on the rafter which is anchored, the stresses in the members of the frame are more severe than when it acts on the free rafter. This should be remembered when designing a roof.

Since we know all the elements of the external forces, the line of loads may be drawn as in Fig. 14b.

Therefore, in order to complete the Stress Diagram we can begin at the top joint of the Frame Diagram where only two members meet. This enables us to find first the point P in the Stress Diagram, then the point Q, and so on.

Note for page 2.—The word *Vector* was used in defining the conditions of equilibrium in frames, consequently it may be as well to define the following terms here:—

Scalar.—A quantity which has no relation to definite direction in space, or which is considered apart from such direction, is called a “Scalar” or “Scalar-Quantity.”

Vector.—A geometrical quantity which is related to a definite direction in space is called a “Vector” or “Vector-Quantity.”

Vector-Quantity.—This requires for its complete determination (1) the magnitude, (2) the direction, and (3) the *sense* to be given.

A vector-quantity may be geometrically represented by a line, if—

- (1) The length of the line represents to scale the magnitude of the quantity.
- (2) The line be placed in the proper direction.
- (3) The proper sense or way be given to the line.

The sense is usually indicated by an arrowhead on the line.

The line itself with its direction and sense is called a *Vector* $\rightarrow$

Suppose that a force of known magnitude acts along a line from P to $\rightarrow Q$; then, the Vector is written down as $\overline{PQ}$, with a bar-line over the two letters P and Q.

Any quantity, whether *scalar* or *vector* (considered as occupying a definite position in space), is said to be *localised*. Thus the mass of a body in a given position is a *localised scalar*, and a force acting on a body at a definite point is a *localised vector*.

Vector Sum.—The sum of a number of vectors is often called the Resultant Vector, and in relation to this resultant the other Vectors are called Components.

To add a number of vectors, place the first anywhere, the beginning of the second to the end of the first, and so on, then the vector from the beginning of the first to the end of the last is the SUM OF THE GIVEN VECTORS (*Henrici and Turner*).

Rotor.—A localised vector is called a *Rotor* (*Clifford*).

* Those who desire to study this subject should read *Vectors and Rotors with Applications*, by Prof. O. Henrici, F.R.S., and G. C. Turner, B.Sc., published by Edward Arnold, London.

LECTURE I.—QUESTIONS.

1. What is a frame or framed structure? Distinguish between the three different kinds of frames.

2. Explain in your own words Bow's method of lettering a system of forces, with two examples.

3. What is meant by the reciprocal of a point, and a pair of reciprocal figures?

4. Explain how you would represent forces in a diagram so as to determine those in each part of a structure, and explain the principles upon which the construction depends.

5. State a rule for determining the kind of stress in a bar.

6. Illustrate and explain how you would find the stresses in a firm quadrilateral frame.

7. Illustrate and explain how you would find the stresses in a firm triangular frame.

8. Illustrate and explain how you would find the stresses in the outline of a Mansard frame.

9. Draw a parallelogram, $ABCD$. The side AB is 2 inches, and the side AD is 3 inches; the angle BAD is 75° . The point E bisects the side CD . There are forces, in the direction DA of 15 lbs.; in the direction AB of 20 lbs.; in the direction EB of 23 lbs.; find the resultant, giving its magnitude, the angle which it makes with AD , and its sense.

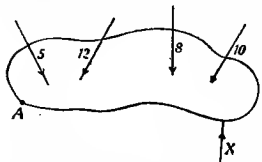
(B. of E. Adv., 1900.)

10. Draw a parallelogram, $ABCD$. The side AB is 2 inches, and the side AD is 3 inches; the angle BAD is 75° . The point E bisects the side CD . There are forces in the direction DA of 15 lbs., in the direction AB of 20 lbs., in the direction EB of 23 lbs. What two forces, acting one along DC and the other passing through the centre of the parallelogram, will produce equilibrium? (B. of E., H., Part I., 1900.)

N.B.—See *Appendices B and C* for other questions and answers. This note refers to all the Lectures.

LECTURE I.—A.M. INST.C.E. EXAM. QUESTIONS.

1. The figure shows forces acting upon a structure in a plane; the direction and position of one supporting force, X , are shown; you are informed that the other acts through the point A . Find these supporting forces.



(I.C.E., Oct., 1898.)

2. The following forces act on a particle situated at the origin of co-ordinates in the plane of the paper:—A force of 5 lbs. making an angle of 30° with the axis of x , 9 lbs. at an angle of 90° , 7 lbs. at 135° , 10 lbs. at 225° , and 3 lbs. at 300° , find graphically the resultant in magnitude and direction. (I.C.E., Oct., 1899.)

3. Write down the conditions of equilibrium for a particle and for a rigid body. (I.C.E., Oct., 1899.)

4. What are the graphical conditions of equilibrium for a system of coplanar forces? (I.C.E., Oct., 1899.)

5. Show how to draw the link polygon for a system of coplanar forces acting on a body. If the force polygon closes and the link polygon* does not, show that the forces acting on the body are equivalent to a couple. (I.C.E., Feb., 1900.)

6. A masonry wall 10 feet in height, with a uniform thickness of 2 feet, is subjected to a horizontal wind-pressure of 28 lbs. per square foot on one side. Taking the weight of the masonry at $1\frac{1}{4}$ cwts. per cubic foot, find the point at which the resultant line of pressure will intersect the base of the rectangular section of the wall. (I.C.E., Feb., 1901.)

N.B.—See Appendices B and C for other questions and answers.

* In dealing with problems in which we wish to find the resultant of a number of forces, we usually require to know the *position*, as well as the *magnitude* and *direction*, of the resultant. And unless all the forces pass through the same point (in which case the resultant will also pass through that point) some separate construction must be used. Such construction is known as the *link and vector polygon construction*, where the vector or *force polygon* alone is sufficient to find the magnitude of the resultant of any number of forces, if the forces meet at a point. But we need also the *link or funicular polygon* to find a point in the line of action of the resultant if the forces do not meet at a point. (For proof, see Prof. O. Henrici's book on *Vectors and Rotors*, published by Edward Arnold, London.) The student is referred to p. 108, Lecture V., Vol. III., for a practical application of the above construction in order to determine the centre of gravity of an area.

LECTURE II.

CONTENTS.—Substituted Frames—King Post Truss—Right-Angled Strut Truss—Roof Truss—Load at an Internal Joint of a Frame—Modified French Truss—Bowstring Truss—Example I.—Notes on Roof Trusses by Mr. J. M. Ringquist—Weight of Roof, Purlins, Rafters, and Covering—Table of Stresses in Members of French Truss—Questions.

Substituted Frames.—The types of frames illustrated in the previous Lecture, although not practical examples, are intended to be substituted for some other actual form in order to determine the reactions therein, since the reactions do not depend upon the form of frame carrying the roofing, but merely on the distribution of the loads. In substituting one of the above frames for a practical one, we must have the joints of the substituted frame coincident with those of the given one. This will be illustrated by the following examples:—

King-Post Truss.—In Fig. 18 we have the Frame Diagram of a king post truss with wind pressure on the right-hand rafter. In this case, we assume both rafters to be anchored to the walls. Therefore, all we know about the elements of the reactions are

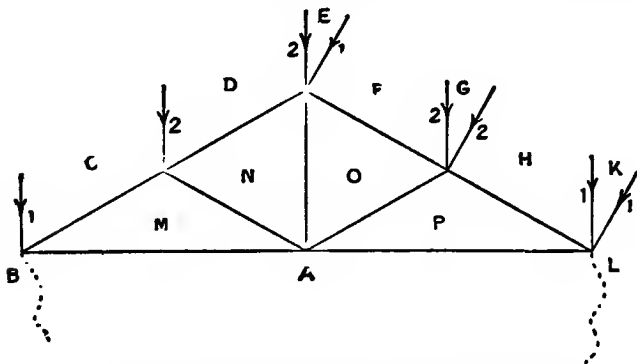


FIG. 18.—FRAME DIAGRAM OF KING POST TRUSS.

their points of application, and that their lines of action are parallel to each other, as well as to the line of action of the resultant of the external forces.

Before we can determine the Stress Diagram for this frame we must first determine the reactions, because more than two bars

meet in each of the joints except the two where the reactions act. Consequently, until we determine all the elements of the reactions we cannot begin at either of these two joints.

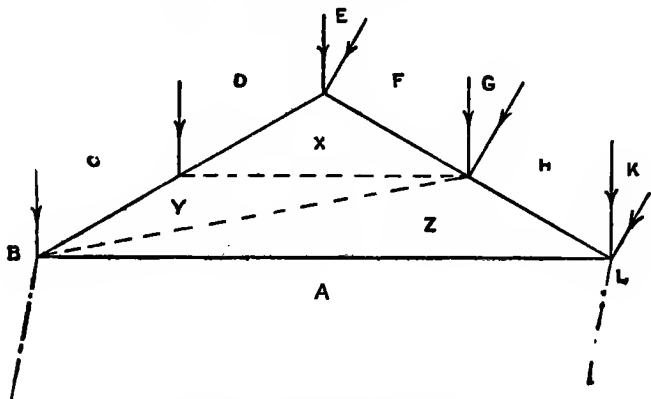


FIG. 19.—SUBSTITUTED FRAME.

In order to determine the reactions, we shall substitute a frame similar to that illustrated in Fig. 15a. This substituted

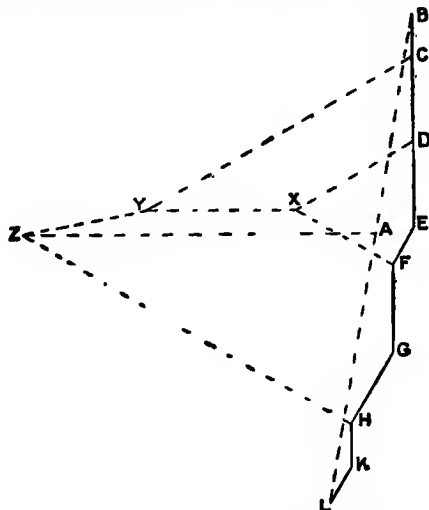


FIG. 20.—STRESS DIAGRAM FOR SUBSTITUTED FRAME.

frame is shown in Fig. 19. In practice, this frame is merely sketched in order to apply the proper letters. The dotted lines are drawn in the Frame Diagram, or the set square is simply made to pass through the requisite joints, and then the lines are drawn parallel thereto in the Stress Diagram. To obtain Fig. 20 we begin by drawing the line of loads. Then, we find the point X, when a line from the point X drawn parallel to the substituted bar X Y, and one from the point C parallel to the rafter O Y fix the point Y. Next we find the point Z. Now, the line of action of the resultant of the external forces is

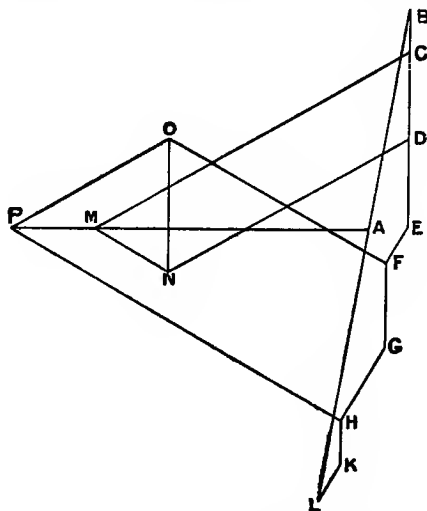


FIG. 21.—STRESS DIAGRAM FOR KING POST TRUSS.

parallel to the line joining L with B. Therefore, the point A must lie on this line since the reactions L A and A B are parallel to each other and to the line of action of this resultant. Consequently, we find the point A by drawing through the point Z a line parallel to the bar Z A so as to intersect L B in the point A. This determines all the remaining elements of the reactions, viz:—

- (1) Their lines of action parallel to L A and A B.
- (2) Their ways from L towards A, and from A towards B.
- (3) Their magnitudes by the number of units of length in the lines L A and A B.

We can now draw the Stress Diagram for the king post truss,

as shown in Fig. 21, from which the particulars for the various members may be determined. Comparing Fig. 21 with Fig. 20, we see that nearly all the lines of Fig. 21 lie along the lines of Fig. 20. In practice we simply draw Fig. 21 on the top of Fig. 20.

This method of a substituted frame introduces fewer errors due to drawing, than the usual method of the funicular polygon (which will be illustrated further on), because we make use of the same joints of the frame for the two Figs. 20 and 21, and the same line of loads.

There is one line in Fig. 21 which will check the accuracy of the Stress Diagram. In drawing the diagram we begin with the point M, and then find the points N, O, and P. The line joining P to H will then be parallel to the rafter PH, if the Stress Diagram is correct.

Right-Angled Strut Truss.—In this frame we have introduced loads at the lower joints as well as roofing weights and wind

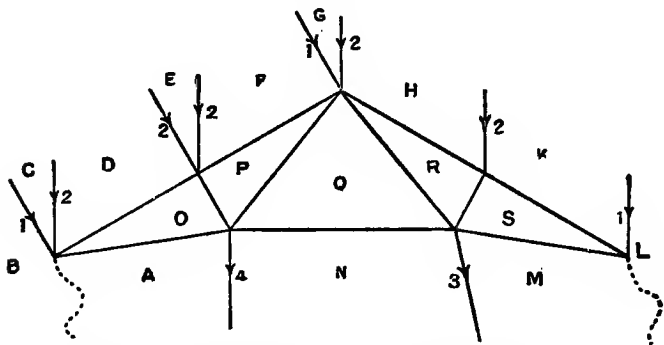


FIG. 22.—FRAME DIAGRAM FOR RIGHT-ANGLED STRUT TRUSS.

pressure. We also assume the two rafters to be anchored to the walls, as indicated by the two dotted curved lines LM and AB.

We must first find the reactions before we can draw the Stress Diagram. In finding the reactions we will substitute the frame which is shown in Fig. 23.

FIRST METHOD OF OBTAINING STRESS DIAGRAM FOR ORIGINAL FRAME.—Produce the lines of action of the loads at the lower joints until they intersect the rafters. These points of intersection are considered as joints in arranging the substituted frame and the lower loads assumed to be acting at these joints

as shown in Fig. 23. The forces fF and Hh in Fig. 23 are the loads NA and MN in Fig. 22 transferred as explained.

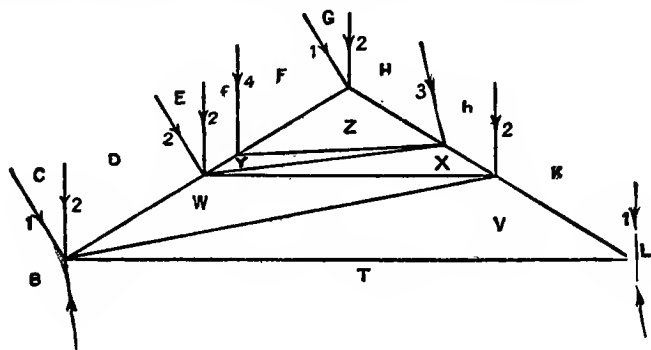


FIG. 23.—SUBSTITUTED FRAME.

The Stress Diagram for the substituted frame is illustrated in Fig. 24 and presents no difficulty requiring explanation. This diagram gives the reactions LT and TB . (See note at the end of this Lecture.)

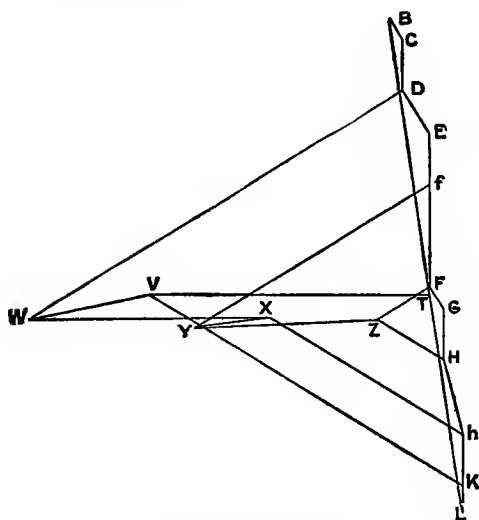


FIG. 24.—STRESS DIAGRAM FOR SUBSTITUTED FRAME.

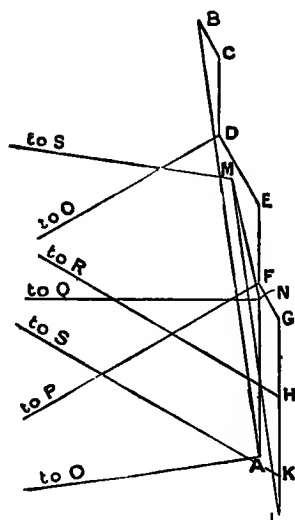


FIG. 25.—STRESS DIAGRAM FOR ORIGINAL FRAME.

In order to draw the Stress Diagram for the original frame which is illustrated in Fig. 25, it is necessary to redraw the line of loads taking them in their order as in Fig. 22. That is, B C,

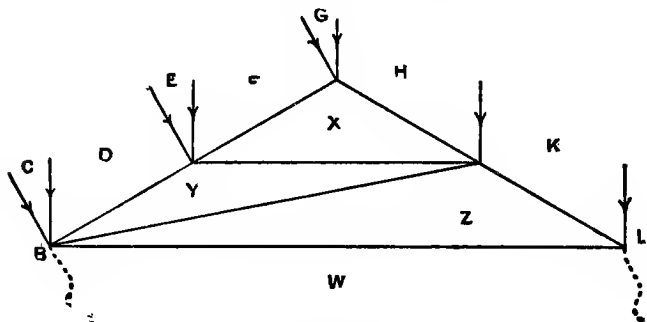


FIG. 26a.—FRAME DIAGRAM.
SUBSTITUTED FRAME FOR TOP JOINT LOADS.

CD, DE, EF, FG, GH, HK, KL, reaction LM, MN, NA and then reaction AB. The drawing of the remaining part of the Stress Diagram calls for no special remark. (The above Stress Diagram is not completed for want of space.)

SECOND METHOD.—Take the top joint and the lower joint loads separately. In Fig. 26a we have the substituted frame for the top joint loads and its Stress Diagram. The Stress Diagram, Fig. 26b, determines the reactions LW and WB due to the loading on the rafters.

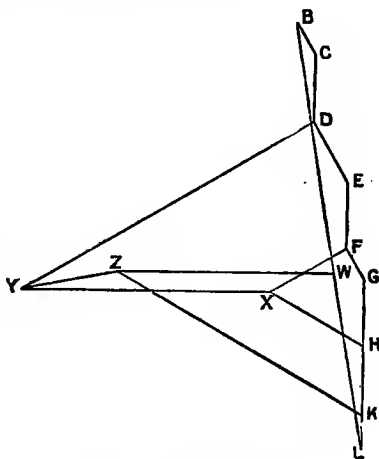


FIG. 26b.—STRESS DIAGRAM.

In Fig. 27 we have a frame similar to the one illustrated in Fig. 14a. The joint A N S is any point in the line of action of the load N A in Fig. 22, and the joint N M R S any point in the line of action of the load M N in Fig. 22. The left and right hand lower joints of Fig. 27 are the rafter ends in Fig. 22

In Fig. 27 the load $T A$ is the reaction $W B$ found in Fig. 26*b* reversed. The loads $A N$ and $N M$ are the loads at the lower

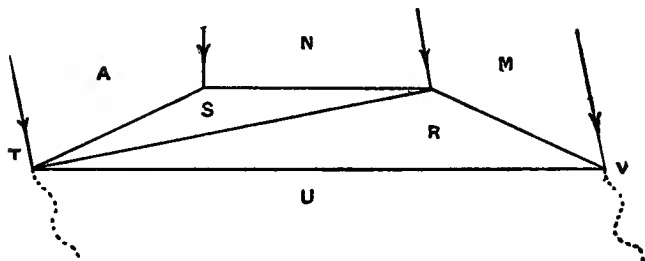


FIG. 27.—SUBSTITUTED FRAME DIAGRAM FOR LOWER JOINT LOADS.

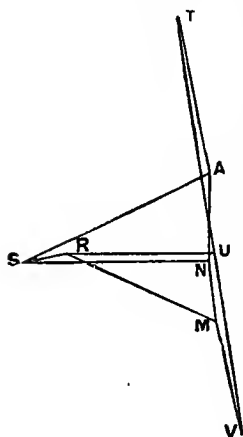


FIG. 28.—STRESS DIAGRAM FOR FRAME IN FIG. 27.

joints in Fig. 22, and the load $M V$ is the reaction $L W$ of Fig. 26*b* reversed. Therefore, if we draw the Stress Diagram for the frame of Fig. 27 we determine the reactions due to all the loads of the original frame of Fig. 22.

In Fig. 28 we have the Stress Diagram for the frame of Fig. 27. The reactions are represented by the lines $V U$ and $U T$. This figure has been drawn to a smaller scale than Fig. 24, but the lines $V U$ and $U T$ of Fig. 28 contain the same number of units as $L T$ and $T B$ of Fig. 24.

**Roof Truss.*—In Fig. 29 we have a frame of a type that will not allow of the Stress Diagram being drawn in a regular manner, but only in a step by step process.

Fig. 30 shows the substituted frame used in order to determine the reactions $L A$ and $A B$, and in Fig. 31 we have the Stress Diagram for both Figs. 29 and 30.

In Fig. 31 we draw first the line of loads, second we find the point O , then $O X$ and $D X$ fix the point X , and $X Y$ and $K Y$ fix the point Y . On drawing $Y A$ parallel to the bar $Y A$, and $L A$ parallel to the line of action of the reaction at the right-hand joint, we determine the reactions $L A$ and $A B$.

The point O is the same in both Stress Diagrams, but we

** See end of this Lecture for criticism of Figs.*

cannot determine the points N and Q until we have found the points M and R. The point M is found by drawing DM

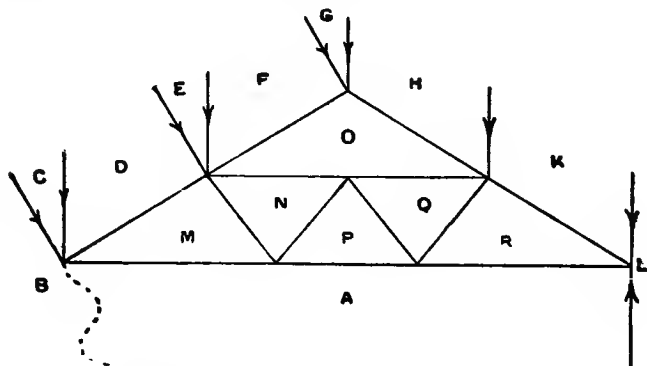


FIG. 29.—FRAME DIAGRAM OF ROOF TRUSS.

parallel to the rafter D M, and A M parallel to the tie-bar A M then M N and N O fix the point N, and similarly for the point

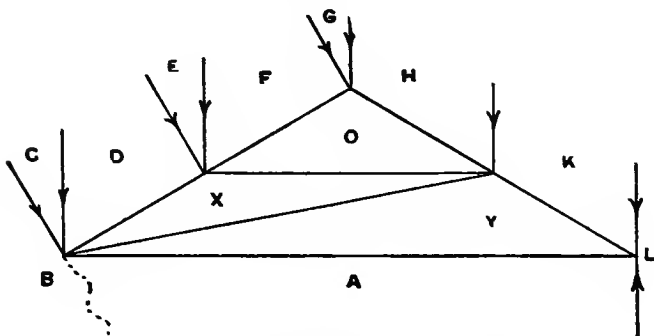


FIG. 30.—SUBSTITUTED FRAME FOR FIG. 29.

Q. Again, although N may be determined, P cannot be fixed until we have found Q.

The line N P or P Q forms a check line.

Note.—The dotted lines of Fig. 31 are the only part of the Stress Diagram for the substituted frame that has not been required for the original frame.

Load at an Internal Joint of a Frame.—In Fig. 32*a* we have a frame the same as the last one, but with a load applied at a joint inside the frame. This load is represented in the Frame Diagram by the small arrow near the letter P, and is applied at the joint OPQT.

*joint where the load is acting and the rafter, and the equivalent force applied at the rafter end of the bar.**

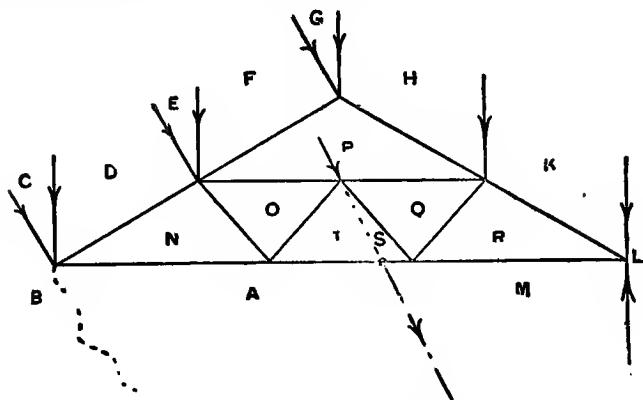


FIG. 32a.—FRAME DIAGRAM.
ROOF TRUSS, WITH LOAD AT INTERNAL JOINT.

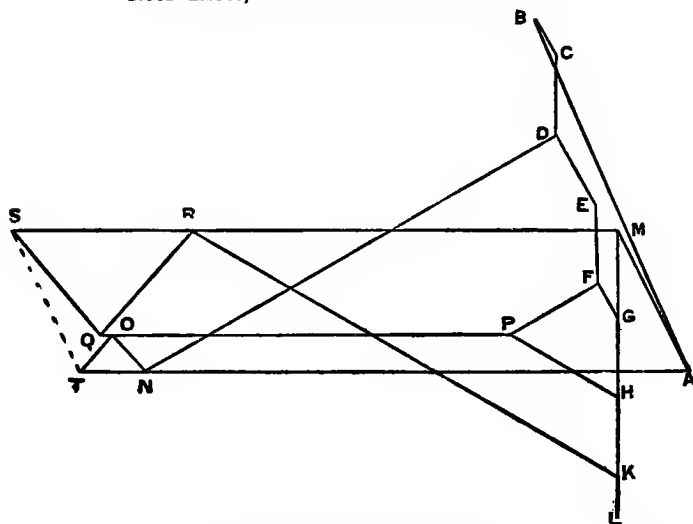


FIG. 32b.—STRESS DIAGRAM.

* The student should work this method as an exercise. Some of the lines of the Stress Diagram will be lowered but the stresses will remain unchanged.

The Frame Diagram, Fig. 32*a*, is loaded similarly to the frame of Fig. 22. The reactions are therefore found in the same way. After the external force polygon has been drawn, the Stress

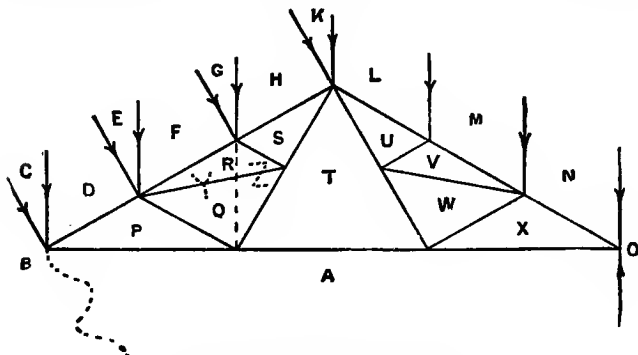


FIG. 33*a*.—FRAME DIAGRAM.

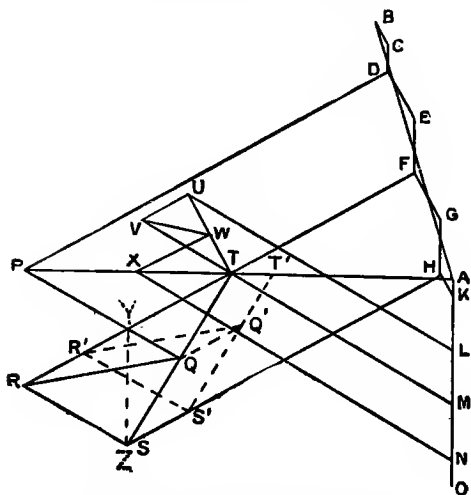


FIG. 33*b*.—STRESS DIAGRAM.

FRAME REQUIRING SPECIAL METHODS FOR SOLUTION.

Diagram may be completed by the same method as that used for the frame of Fig. 29.

Referring to the Stress Diagram of Fig. 32*a*, we see that the

bar TS exerts the same pull at the joint TSMA as the force MA. These two therefore balance each other, and we are left with the pull which the bar ST exerts on the joint OPQT. This pull ST, on the joint OPQT, is identical in all its elements with the load; and therefore, the stresses in the members of the frame will be identical with those due to the original load.

Modified French Truss.—This Truss is illustrated in Fig. 33a, and presents some difficulties in its solution. We first determine the reactions as already explained and draw the external force polygon as in Fig. 33b. Secondly, we draw DP and AP parallel to the bars DP and AP respectively. This fixes the point P. But, although we know the point P, we can get neither Q nor R, nor any other point but X. This point X, however, does not help us, because we can proceed no further by aid thereof with the Stress Diagram.

FIRST METHOD OF OBTAINING THE STRESS DIAGRAM.—We know that the point R lies on a line drawn through F parallel to the bar FR and that S lies on a line drawn through H parallel to the bar HS. Now, assume a point R' anywhere on the line FR and draw R'S' parallel to the bar RS and HS' parallel to the bar HS. This fixes the point S'. Then S'T' and A T' fix the point T', and R' Q' and T' Q' fix the point Q'. Next move the figure R'S'Q' parallel to itself keeping R' on the line FR until Q' lies on a line drawn through P parallel to the bar PQ. This is done by drawing Q'Q parallel to FR so as to intersect PQ in Q. This determines the point Q. Then QS and HS fix S and QR and FR fix R and so on for the other points.

SECOND METHOD.—Substitute the bar YZ (as shown by the dotted line in the Frame Diagram, Fig. 33a) for the two bars QR and RS. This bar transfers the action of the loads at the joint GHSRF to the joint PQT A; and therefore, the stress in TA will be unaffected. If the bar HS had been divided and similarly braced, then a bar from that joint to the joint PQT A would enable a solution to be found.

In the Stress Diagram, Fig. 33b, we begin by finding the point P, then the points Y, Z, and T respectively. Having found the point T we can then proceed to find the other points in the same way in the previous cases.

THIRD METHOD.—First, find the stress in the bar TA, by taking one of the sections of the truss and thus obtain the resultant of the loads and the reaction of the wall on that section. Second, ascertain what stress in TA combined with the reaction of the other section of the truss on the apex will

balance this resultant. Fig. 34 represents the right-hand section of the Truss of Fig. 33a. There is no wind pressure on this side. The loading consists of $\frac{1}{2}$ KL, LM, MN and NO in Fig. 33a. The resultant $K'O$, Fig. 34, of these four loads passes through the centre of the rafter since NO is equal to $\frac{1}{2}$ KL and LM is equal to MN.

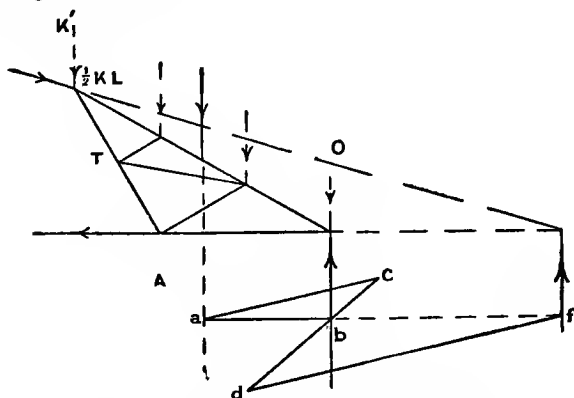


FIG. 34.—LOADS ON RIGHT-HAND SECTION.

We have now to find the resultant of $K'O$ and the reaction OA . Draw the line ab anywhere cutting the lines of action of the forces $K'O$ and OA as shown in Fig. 34. Through b draw cd at any angle to ab . Make bc represent to scale the force $K'O$ and cd to the same scale the reaction OA . Then join a with c and through d draw df parallel to ac , so as to intersect ab produced in f . Then f is a point in the line of action of the resultant of the forces $K'O$ and OA . Its line of action is parallel to the lines of action of the forces $K'O$ and OA , and its magnitude is equal to their difference—that is, $K'A$ in Fig. 35.

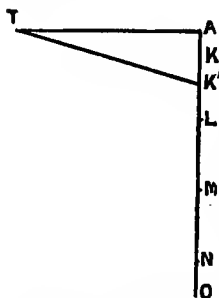


FIG. 35.—TRIANGLE OF FORCES ON SECTION.

Next produce the centre line of the bar TA in Fig. 34 to intersect the line of action of $K'A$, the resultant of the two forces $K'O$ and OA . Then since the resultant $K'A$, the action of the stress in the bar TA , and the action TK' (i.e., the reaction of the left-hand section on the apex of the frame) form a system of

forces in equilibrium. The line of action of the force TK' must pass through the apex and the intersection of TA and

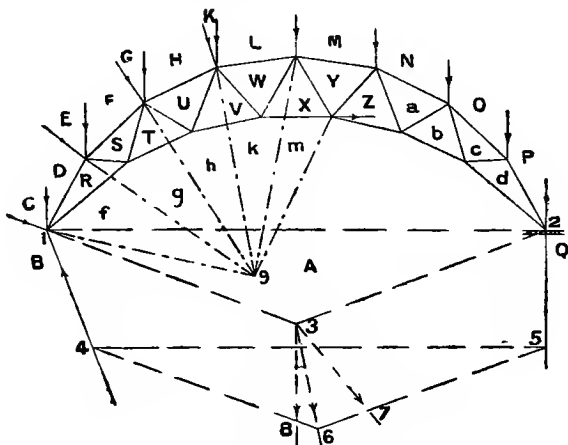


FIG. 36a.—FRAME DIAGRAM. BOWSTRING TRUSS.

K' A, consequently an application of the triangle of forces will give the value of the stress in **T A**. This is shown in Fig. 35, where **K'** is the centre of **K L** and the other points are points in the line of loads as in Fig. 33*b*. When this is known, the Stress Diagram can be completed.

Bowstring Truss.—There is no difficulty in drawing the Stress Diagram for this truss, but if we commence in the usual manner by drawing DR and AR , by the time we get to the other side, the finishing line would most probably not be parallel to its corresponding bar in the Frame Diagram. This is due to the short

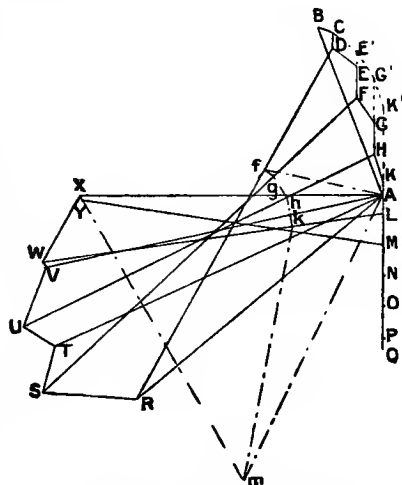


FIG. 36b.—STRESS DIAGRAM.

length of the bars R S, S T, &c., in the Frame Diagram and to the stresses in them being large compared to the loading on the roof as shown by the Stress Diagram.

We get a very much better diagram by determining the stress in one of the centre ties—*e.g.*, A X, by aid of a supplementary frame. Therefore, in order to find the reactions, the wind pressure may be supposed to act on a surface tangential to the curve of the roof at the joints; the length of the surface being equal to the sum of the two half bays on each side of the joint. The wind pressure at each joint will act along the radial line at the joint, and therefore the resultant of the wind pressures must pass through the centre of the outer curved flange.

The point 3 in the Frame Diagram, Fig. 36*a*, is the centre of the outer curved flange. This is a point in the line of action of the resultant wind pressure. This line is parallel to the line joining B with K' in the Stress Diagram, and is represented by the line 3—7 in the Frame Diagram. B C, C E', E' G', and G' K' represent the wind pressures B C, D E, F G and H K and therefore B K' is the resultant in magnitude and is parallel to its line of action.

If the roofing is uniform, the centre of the curve of the outer flange will be a point in the line of action of the resultant load. Therefore, the resultant of the wind pressure and of the roofing weight will also pass through the point 3. The line of action of this resultant will be parallel to the line joining B with Q in the Stress Diagram—*i.e.*, along the line 3—6 in the Frame Diagram. The Truss is in equilibrium under the resultant load acting along the line 3—6. The line of action of the right-hand reaction is known, and the point of application of the left-hand reaction is also known. These three forces must pass through one point. Therefore the line joining the point 1 with the point where the line 3—6 cuts the line 2—5 will give the line of action of the left-hand reaction. But as the point of intersection of the lines 3—6 and 2—5 would be far off the paper we use the following construction:—Join the point 1 with 2, 2 with 3, and 3 with 1; then take any point 5 in the line of action of the right-hand reaction and draw 5—6 parallel to 2—3. Then draw 6—4 parallel to 3—1 and 5—4 parallel to 2—1 so as to intersect 6—4 in the point 4. If the point 1 be joined with the point 4 the line 1—4 will pass through the point of intersection of the line 3—6 with the line 2—5. The line 1—4 is therefore the line of action of the left-hand reaction.

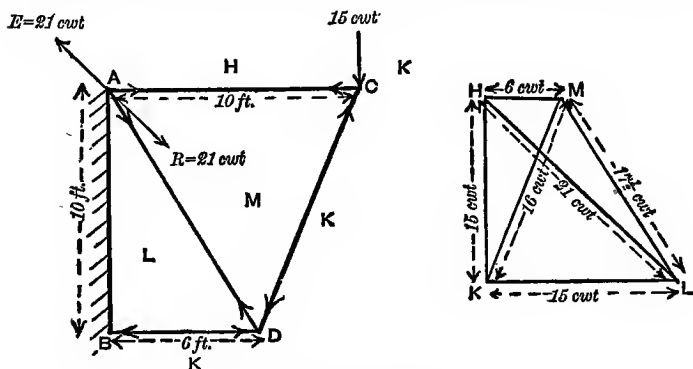
In the Stress Diagram, Fig. 36*b*, draw a line through the point B parallel to the line 1—4, so as to intersect the line Q A in the point A. This determines the reactions and enables us to proceed with the Stress Diagram.

We shall first determine the stress in XA . Assume a point 9 anywhere and connect it as indicated by the chain dotted lines in the Frame Diagram. This point 9, when connected with the joints of the outer flange, forms a supplementary frame in equilibrium under the following forces:—

- (1) The wind pressure.
- (2) The loads CD , EF , GH , KL and LM .
- (3) The reaction AB .
- (4) The action of the stress MY on the joint $LMYXW$.
- (5) The action of the stresses in YZ and ZA on the joint $XYZA$.

In the meantime, the internal bars RA , RS , ST , TA , on to WX and XA are left out of account.

In the Stress Diagram, Fig. 36*b*, we begin by drawing Af parallel to the supplementary bar Af and Df parallel to the bar Df . This fixes the point f . Then fg and Fg give the point g and so on until the point m is obtained. Now draw mY parallel to the bar mY (which is coincident with the bar XY) and MY parallel to the bar MY this fixes the point Y . In a similar way YX and AX fix the point X , when the Stress Diagram may then be finished in the usual way. This method gives



TO ILLUSTRATE EXAMPLE I.

a more accurate estimate of the several stresses than by following the usual direct plan, as explained at the beginning of this example. Moreover, this construction is perfectly general in its application and may be used to determine the stress in any one bar of a frame.

EXAMPLE I.— A is a point in a wall 10 feet vertically over another point B . From A and B there project two horizontal

bars, A C, B D (the former being 10 feet and the latter 6 feet long), and D is joined by two bars with A and C. If a weight of 15 cwts. be hung from C, find the stresses on all the bars, and show which are in tension. Find also the resultant stress on the point A. You may neglect the weights of the bars.

ANSWER.—In the figure we have denoted the spaces by letters according to Bow's notation. To obtain the Stress Diagram we must draw H K parallel to the force H K and 15 units long. Then make H M parallel to the bar H M and K M to the bar K M. This gives us the point M. M L parallel to the bar M L, and K L to the bar K L, fix the point L. H L, when joined, gives the reaction at the joint A, and the other lines the stresses in the bars. Their values are marked on the figure. A C and A D are in tension, while C D and B D are in compression.

Note to Fig. 24.—Mr. Vowell says—"In regard to the general method of the 'Substituted Truss or Frame,' the Funicular Polygon appears simpler. (See Index for page where this is explained.) In the 'Substituted Frame' method, one is obliged to get the point T through several short lines, thus making accuracy difficult, whereas, by the Funicular Polygon Method you can always deal with the resultant wind pressure *only*, and split it up by the shorter 'Culman's Method.'" (See Appendix E for an explanation of this Method.)

Notes by Mr. J. M. Ringquist, Chief Draughtsman, Stockton-on-Tees.

(1) **Roof Truss.**—In Fig. 29, on p. 27, is shown a truss which is neither efficient nor economical, since the compression members D M, F O, O H, and R K are not in any way reduced in length by the bracing bars M N, N P, P Q, and Q R.

(2) In the roof truss illustrated by Fig. 32*a*, on p. 29, the student will notice that no member of the truss should be loaded between the joints, as shown at the member S A, if it can be avoided.

The reason for this is, that such loading causes the member to be treated as a beam loaded at the point of application of the weight. Hence, instead of determining the sections of the rafters as in the case of compression members only, you have to combine the compression and bending stresses.

(3) **Iron King Post Truss.**—In all frame diagrams, as shown on p. 42, it is a general custom to make the centre lines of gravity of the members to intersect in all heavy roofs. But, for smaller roofs, made of T or L rafters, the "scribe lines" or "back-lining" intersect—i.e., the centre line of rivets, which, of course, is not the same as the centre of gravity lines.

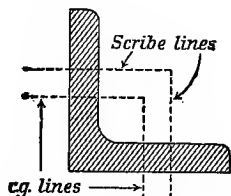


FIG. 36c.—SHOWING CENTRE OF GRAVITY (C.G.) AND SCRIBE LINES.

In general practice a truss would not be made as shown by the two inclined full lines from the tie-piece at A in Fig. 37*a*, because the excess in stress due to wind pressure acting on one end side of the tie-piece at the foot of the king rod would tend to tilt it. This is not permissible in roof trusses. The usual method now, is to take the bracing bars into the centre at A, as indicated by the dotted lines on Fig. 37*a*, which avoids the use of a special tie-foot for the king rod, like that shown by Fig. 38.

Weight of Roof, Purlins, Rafters, and Covering.—The dead load upon the roof of any building consists of—(1) The weight of the roof principals or trusses. (2) The purlins.* (3) The rafters with the roof-covering.

The trusses and purlins are sometimes dispensed with in roofs of small span.

The principal rafters are straight, and abut against each other at the peak or top of the principal, whilst they are prevented from spreading at their lower points or heels by tie-rods. When the rafters are made of iron, tee, rail, and both single and double channel sections, bulb-tee and rolled I iron girders, all prove very serviceable. If the rafters are made of timber, their cross-section is rectangular in form; and, for the sake of economy, combined with general appearance, they are made to taper uniformly from the heel to the peak or top of the principal. The weight of the roof principal or truss may be estimated from the known weight of similar roofs for the purposes of calculation, and the cross-sectional areas of the members thus determined. Afterwards, the correct weight may be ascertained and the truss stresses re-calculated for the corrected weight.

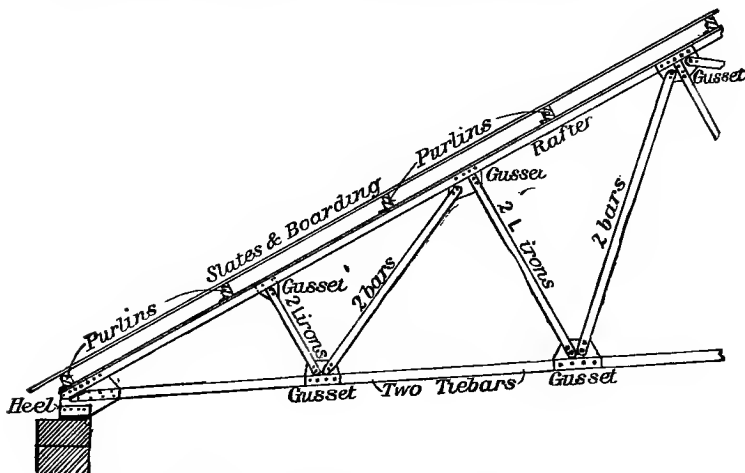


FIG. 37.—ROOF TRUSS, SHOWING PURLINS, RAFTERS, AND COVERING.

The weights of the purlins may be ascertained without much trouble, as they consist of rectangular timber beams, angles, tees, or rolled steel girders. In large roofs the purlins may be lattice girders or truss beams. The purlins are spaced at intervals along the principal rafters, about 8 to 10 feet, but varying with the type of roof. The spacing of the purlins is also regulated by the nature of the covering, and the dimensions of the purlins are governed by the distance between the main rafters. These

* The purlins are the longitudinal beams carrying the load between the trusses and transmitting the loads to the principals.

purlins are sometimes fixed to the top or sides of the rafters by brackets, or lie between them in shoes made of cast iron. When the distance between the rafters is so great as to cause an undue deflection of the purlins, these latter must be trussed.

The junction of the rafters at the peak or top of the principal is made by means of a casting or wrought-iron plates. The heel of the principal rafters are fitted into cast-iron skew-backs, or are fixed between wrought-iron angle-brackets, and rest either directly upon the wall or upon a wall-plate. (*See Fig. 14a, Lecture I.*)

When the span exceeds 50 feet, allowance must be made for the alterations in length due to the changes of temperature. This may be effected by interposing a set of rollers between the skew-back and wall-plate at one heel, as indicated by the figure previously referred to, or by fixing one heel to the wall and allowing the skew-back at the opposite heel to slide freely over a wall-plate. If one end of the principal is fixed, and the other end is free to move on expansion rollers, the principal will tend to expand or contract as the temperature increases or diminishes. Motion takes place as soon as the forces due to the changes of temperature are sufficient to overcome the frictional resistance of the expansion rollers. When the frictional resistances just balance the forces due to temperature, the whole of the horizontal component of the normal wind pressure on the roof must be resisted by the fixed end, since the roller end can only supply a vertical reaction.

The sheathing boards and final metal or slate covering are fastened upon the purlins by means of nails.

TABLE OF STRESSES IN MEMBERS OF FRENCH TRUSS.

(*Lecture II., p. 41.*)

To illustrate the kind of table which students should draw up in answering such questions on structures as 10 and 11 of this lecture.

Members.	Stress in Tons.	Members.	Stress in Tons.
AJ	+ 11	NM	- 1·3
JI	- 9·5	MP	- 4·6
BK	+ 10·4	DO	+ 9
KL	- 1·3	OP	- 6
LI	- 8·2	PI	- 4
CN	+ 9·8		

NOTE.—The (+) sign indicates, that the member is in compression, whilst the (-) sign shows that the member is in tension. As the loading is symmetrical on each half of the truss, then the magnitudes of the stresses in the members not given are identical with those stated in the table. But, if wind pressure is considered as acting on one side of the truss, then the table of stresses must include every member of the truss in every instance.

LECTURE II.—QUESTIONS.

1. A king post truss, whose height is one-fourth of its span, is loaded at the joints with vertical loads of 15, 30, and 45 units respectively. Determine the nature and amount of the stresses in each member of the frame. (S. & A. Adv. Exam., 1896.)

2. A roof of 28 feet span, height 7 feet, rests on king-post trusses, 10 feet apart. The weight of the roof is 20 lbs. per square foot. Find the stresses on each part.

3. If the above roof has a horizontal wind pressure of 40 lbs. per square foot on one side, find the stresses on each part.

4. A roof of the form shown in Fig. 22, is 40 feet span and 10 feet high. The horizontal tie-bar is 8 feet below the vertex. Find the stresses in each part when loaded with 2 tons at each joint.

5. If, in the previous question, the maximum wind pressure on one side be 2 tons on each bay, find the stresses on all the bars.

6. Suppose both ends of the roof truss in Fig. 29 are anchored, and that in the substituted frame the bar XY slopes in the opposite direction to that in Fig. 30; find the reactions and the stresses in the roof truss.

7. Work out the stresses for Fig. 32a by the method referred to in the second note.

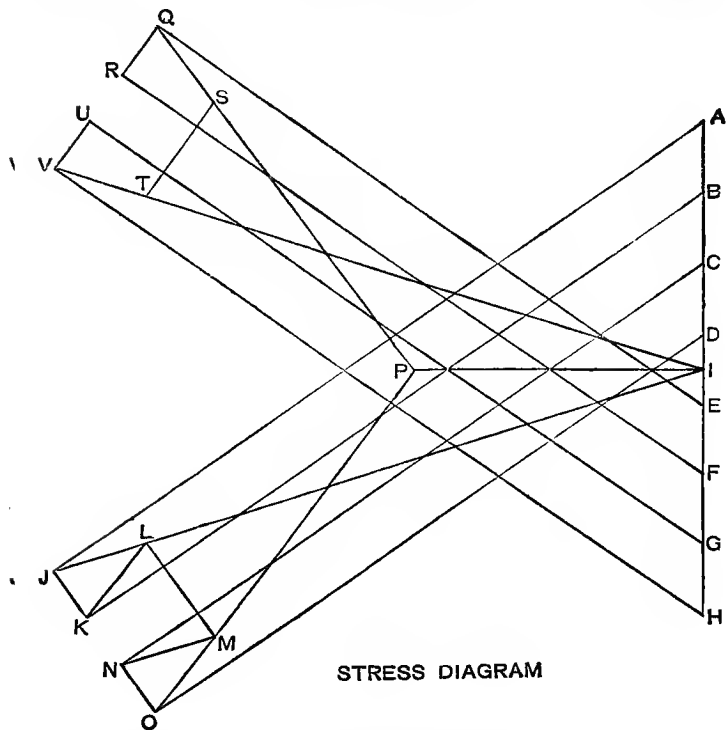
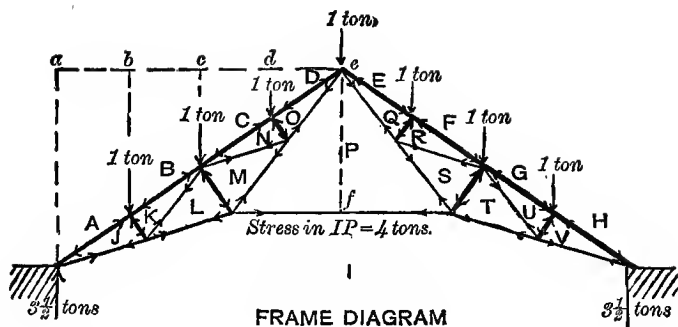
8. Suppose both ends of the modified French struss in Fig. 33a are anchored, and that the substituted bar YZ lies across the spaces V and W ; find the reactions and stress (1) neglecting wind pressure, (2) when wind pressure is taken into account.

9. Find the stresses in the bowstring truss, shown in Fig. 36a, when both the ends are anchored (1) without wind pressure, (2) when wind pressure is taken into account.

10. The following figures give the Frame and Stress Diagrams for a French Truss. Verify the Stress Diagram and redraw it in the manner explained in the text.

11. Draw the Stress Diagram when there is a wind pressure of 4 tons on the left-hand slope, assuming both sides of the roof to be fixed to the walls.

N.B.—See Appendices B and C for other questions and answers.



ILLUSTRATIONS FOR QUESTIONS 10 AND 11.

(See Table of Stresses in Members of French Truss, given at p. 39.)

LECTURE III.

CONTENTS.—Deficient Frames—Iron King Post Truss—Queen Post Frame—Solution of the Second Method—Solution of the Third Method—Solution of the Fifth Method—Yieldingness—Recent Framed Structures and Practical Application of the King Rod or “English” Truss—Questions.

Deficient Frames.—Iron King Post Truss.—In practice the foot of the king rod NO is made virtually solid, as shown by the full lines in Fig. 37a. If the short bars NA and OA are made freely

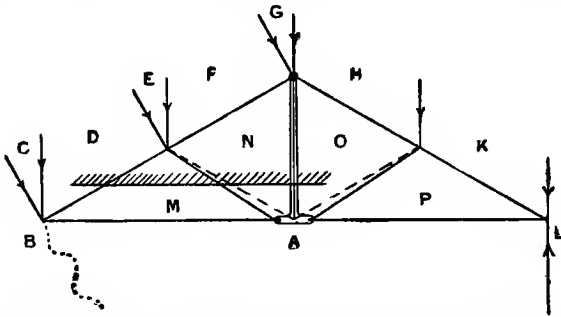


FIG. 37a.—FRAME DIAGRAM FOR DEFICIENT KING POST TRUSS.*

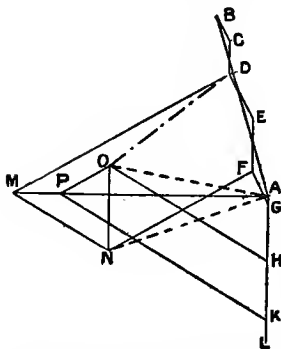


FIG. 37b.—STRESS DIAGRAM FOR DEFICIENT KING POST TRUSS.

jointed at NOA , it will be evident that the spaces N and O would change their shape if the loads at the centre of the rafters were unequal. This change of shape is resisted by making the joint NOA rigid.

Since the bars NA and OA are very short compared with the other members, we can draw the Stress Diagram as if the frame were made as shown by the dotted lines in Fig. 37a. The full lines in Fig. 37b, which is the Stress Diagram of Fig. 37a, are drawn on the above assumption.

* See end of previous Lecture for criticism of Fig. 37a.

If a line of section be drawn, beginning in one space of a Frame Diagram and ending in another, so as to pass through a joint or cross two or more bars, then the line joining the points in the Stress Diagram named after the beginning and end spaces gives the resultant of the stresses in all the bars meeting in or crossing that line.

The student can easily verify this by referring to the Stress Diagram.

In Fig. 37*a* a shaded line is shown beginning in the space D and ending in the space O. Then in Fig. 37*b* the chain dotted line DO is parallel to the line of action of the resultant of the stresses in the bars DM, MN, and NO. The length of the line DO gives the magnitude, and from D to O the way of the resultant with respect to the top side of this section. A point in the line of action of this resultant may be found by drawing a line through the joint DEFNM parallel to the line joining D with N in the stress diagram to cut the bar NO produced. The line DN is the resultant of the stresses DM and MN, which must pass through the joint DEFNM. The dotted line OA gives the resultant of the stress actions in the bars OP and PA on the imaginary joint NOA. Therefore, since the bar OA is very short, the force acting on the pin at the end of this bar will be approximately represented by the elements found from the line OA. Similarly, the dotted line NA gives the elements of the force acting at the end of the short bar NA.

The forces acting at the foot of the King Rod are represented by Fig. 38. These forces produce bending and tension in the parts OA, AN, and NO.

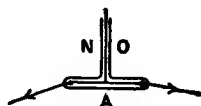


FIG. 38.—FOOT OF KING ROD.

Queen Post Frame.—A Queen Post Frame is represented in its normal position by the solid lines in Fig. 39. If the frame be

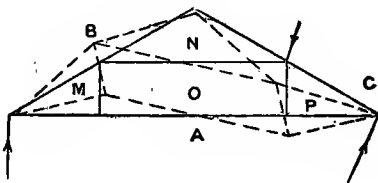


FIG. 39.—DISTORTION OF A FREELY JOINTED QUEEN POST FRAME.

freely jointed, it would be deformed into the shape represented by the dotted lines by a single force BC applied as shown at the joint $NBCPO$.

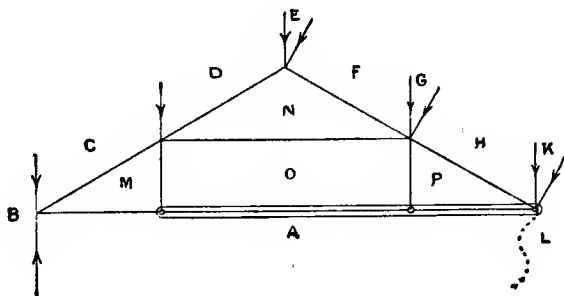


FIG. 40a.—FRAME DIAGRAM.

QUEEN POST FRAME, WITH PART OF TIE-ROD MADE CONTINUOUS.

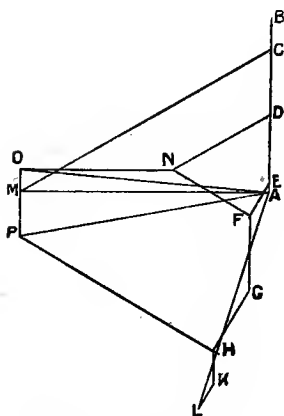


FIG. 40b.—STRESS DIAGRAM.

This change of shape may be resisted in several ways, such as the following:—

- (1) By a diagonal in the central parallelogram. This diagonal would have to stand push if the wind caught the frame on one rafter, and pull if the wind pressure were on the other; or the stresses might be due to snow. It is the usual practice to put two diagonal ties in the parallelogram, so that when a push comes on one diagonal the other receives it as a pull. In drawing the Stress Diagram for such a

frame, if a push comes on one of the ties, we omit that bar and take the other.

- (2) By making the bar continuous between the joints MOA and $PHKL$, and therefore able to resist being bent into the dotted form shown in Fig. 39.

- (3) By making the whole tie-beam continuous. This causes the frame to become redundant; *i.e.*, it may be self stressed, by having the bars *MO* and *OP* of unequal length, or badly fitted.
- (4) By making one rafter continuous.
- (5) By making the rafters and tie-beam continuous. This is the usual form in actual practice and causes the frame to become redundant.

Solution of the Second Method.—The reactions are ascertained by a Substituted Frame as already explained. In the Stress Diagram, Fig. 40*b*, we begin by drawing *DN* and *FN*; *CM* and *AM*; *NO* and *MO*; *OP* and *HP* all parallel to their respective bars. Then *P* and *O* joined with *A* give the finishing lines of the Stress Diagram. The lines *AP* and *AO* are not parallel to the bars *AP* and *AO*. This indicates that there is bending in the continuous part of the tie-rod.

In Fig. 41, the forces are shown acting on the part of the tie-beam which is continuous. The vertical components of the forces *AO* and *AP* produce bending in the bar, while the horizontal components produce tension.



FIG. 41.—THE CONTINUOUS PART OF TIE-BEAM.

Solution of the Third Method.—Having found the reactions and drawn the External Force Polygon, as in Fig. 42*b*, we can then find the point *N*. We observe that *O* must lie on the line *NO*, which is drawn parallel to the bar *NO*; *M* must lie on the line *DM* drawn parallel to the bar *DM*, and *P* on the line *KP* drawn parallel to the bar *KP*.

On reference to Fig. 39 we see, that so long as the rafter ends always remain in the same horizontal line, the joint *OPA* must go down as much below the horizontal line as the joint *MOA* goes above it. Therefore, if the tie-beam is equally rigid along its length, the push required to distort it at the joint *OPA* must be equal to the pull distorting it at the joint *MOA*—that is, *OP* must be equal in length to *MO* in the Stress Diagram, Fig. 42*b*. If the tie-beam be unequally rigid, then the push and pull will be in proportion to the rigidity at the joints *OPA* and *MOA* in Fig. 39. In Fig. 42*a* the distorting force is on the left-hand rafter, and therefore the joint *MOA* will go down; consequently *MO* is subjected to push stress.

We can now proceed with the Stress Diagram in Fig. 42*b*.

Since MO is equal in length to OP , P and M must lie where the line DM intersects the line KP . Again, OP and MO are parallel to the bars OP and MO respectively, and NO is

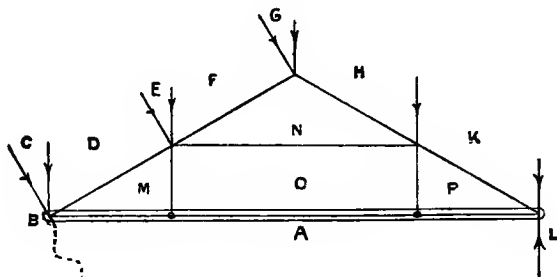


FIG. 42a.—FRAME DIAGRAM.

QUEEN POST FRAME, WITH CONTINUOUS TIE-BEAM.

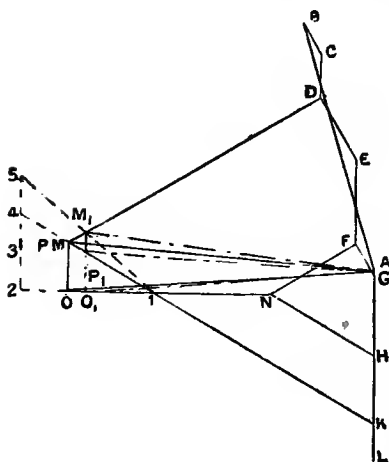


FIG. 42b.—STRESS DIAGRAM.

The horizontal components of the forces AM and PA produce tension in the tie-beam.

Now, suppose the rigidity of the tie-beam at the joint POA to be $\frac{2}{3}$ of its rigidity at the joint MOA , then O_1P_1 must equal $\frac{2}{3}$ of O_1M_1 . We must remember that the joint OPA is always as

parallel to the bar NO . This fixes the point O . Joining the point PM and the point O with A we complete the stress diagram.

The forces acting on the tie-beam are illustrated by Fig. 43. The force OP and the vertical component of PA constitute a couple tending to produce clockwise rotation. The force MO and the vertical component of AM form another couple of equal moment, and also produce clockwise rotation. These two couples bend the beam, as indicated in Fig. 39.

much above as MOA is below the horizontal line. A construction to determine M_1 , P_1 , and O_1 is shown by the dotted lines in Fig. 42b. The point 2 is taken anywhere in the line NO . The line 2—5 is drawn perpendicular to line NO , and the line KP is produced to cut the line 2—5 in the point 4. Then the length 2—4 must be to the length 2—5 as the rigidities at the joints. In other words, the line 2—5 is three when 2—4 is two, and therefore 4—5 is equal in length to half of 2—4. Now join point 5 with point 1. This line cuts the line DM in the point M_1 , and by drawing M_1O_1 parallel to the bar MO , and N_1O_1 parallel to the bar NO , we obtain the point O_1 .

The finishing lines of the Stress Diagram, Fig. 42b, are obtained by joining M_1 , P_1 , and O_1 with the point A , and are shown by the chain dotted lines.

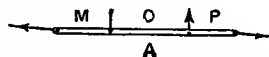


FIG. 43.—FORCES ACTING ON CONTINUOUS TIE-BEAM.

Solution of the Fifth Method.—In this arrangement of bars (Fig. 43a) if the joint $FGHPON$ descends through a small distance (say 1 inch) then the joint OPA of the tie-beam will descend 1 inch, the joint AMO will go up 1 inch and the joint $MCDNO$ will rise 1 inch. Now, all this will take place irrespective of the rafters and tie-beam being of equal or of unequal yieldingness.

Yieldingness.—Two springs are of equal yieldingness, when they stretch through the same amount under equal loads.

One spring would have a yieldingness of three times another, if the first extended three times the amount that the second stretched under the same load.

Further, if two springs of equal yieldingness are attached to the same load, so that they each extend through the same amount; then each spring will carry one half of that load. But, if two springs of unequal yieldingness are attached to the same load, so that they each extend through the same amount; they will each carry a share of the load *inversely* proportional to their yieldingness. Suppose we have two springs, the first one stretches say 1 inch under a load of 3 lbs., while the second one extends 1 inch under 1 lb.; then, if these two springs are set to carry a load of 4 lbs., they will each extend 1 inch and the first spring will carry 3 out of the 4 lbs., while the second will carry the remaining 1 lb.

The above remarks apply equally to bars supporting a load between them, whether they are under a similar kind of stress or not. For example, suppose a beam is jointed to a rod attached

to a right point above it; then, their yieldingness would be measured by the amounts they would each come down under

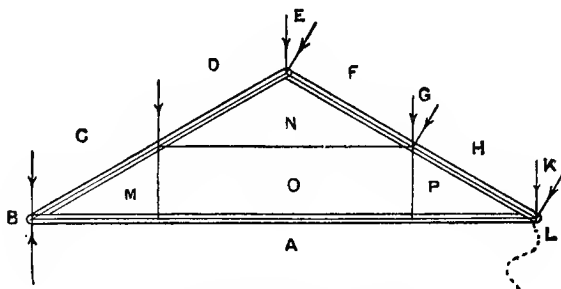


FIG. 43a.—FRAME DIAGRAM.

QUEEN POST FRAME, WITH RAFTERS AND TIE-BEAM CONTINUOUS.

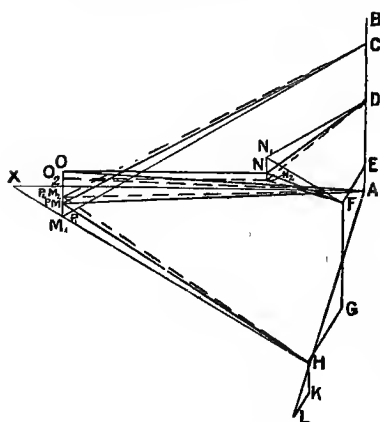


FIG. 43b.—STRESS DIAGRAM.

the same load, as applied to each separately at the point where they are jointed to each other.

Referring to the Frame Diagram, Fig. 43a, we shall assume in the first place, that the yieldingness of the rafter at the centre in a vertical direction, is the same as the yieldingness of the tie-beam at the joint O P A, also in a vertical direction.* Therefore, whatever is the amount of the vertical component of the distorting force, they will each be subjected to the same stress.

Before we can do anything to the Stress Diagram, Fig. 43b, we must first find what amount of the distorting force passes into M O and O P in Fig. 43a, on the assumption that the rafters are freely jointed at their centres.

Figs. 44a and 44b show how this is done. In the Frame Diagram, the force Q G is the difference between the loads C D

* This does not mean that the rafter and the beam have equal rigidity.

the tie-beam. Then O_2M_2 will have the smaller length as its value, if the yieldingness of the rafter is the smaller; and O_2M_2 will have the larger length of SU if the yieldingness of the tie-beam is the smaller. For example, let the tie-beam be twice as yielding as the rafter. Then divide SU in the proportion of 2 to 1—i.e., into three equal parts—and make O_2M_2 equal to one of the three parts; keeping in mind, that O_2 is as much above AX as M_2 is below it.

Recent Framed Structures and Practical Application of the King-Rod or "English" Truss.—The following four figures, 46, 47, 48, and 49, will prove of interest to the student as illustrating the latest type of framed structure adopted for roofs. These were designed and made by Sir William Arrol & Co., Limited, for the extension of their own works in Glasgow. The author is indebted to the firm for kindly supplying the drawings from which these figures were specially made by the publishers of this book.

This form of roof, as shown by the sectional elevation is suitable for spans up to 60 or 70 feet, and has been extensively adopted for railway stations. The verticals are made of angle iron 3 inches $\times$ 2½ inches $\times$ ¼ inch, whilst the inclined members are made of two angle irons of the same dimensions. The purlin at the top and at the bottom of the rafter is formed of one angle iron 3½ inches $\times$ 3 inches $\times$ ⅝ inch; whilst the double purlins, which are placed at intermediate positions along the rafters, are formed of angle irons 3½ inches $\times$ 3 inches $\times$ ¼ inch.

Wind bracing is effected by horizontal angle irons 3 inches $\times$ 3 inches $\times$ ⅜ inch, connected diagonally across the span between each second column A at the level of the roof truss.

It will be observed, that this design of structure gives a complete clear space from the floor to the roof truss. It affords perfect freedom for the manipulation and assembling by the electric travelling crane of the various parts which may have to be constructed under the main building.

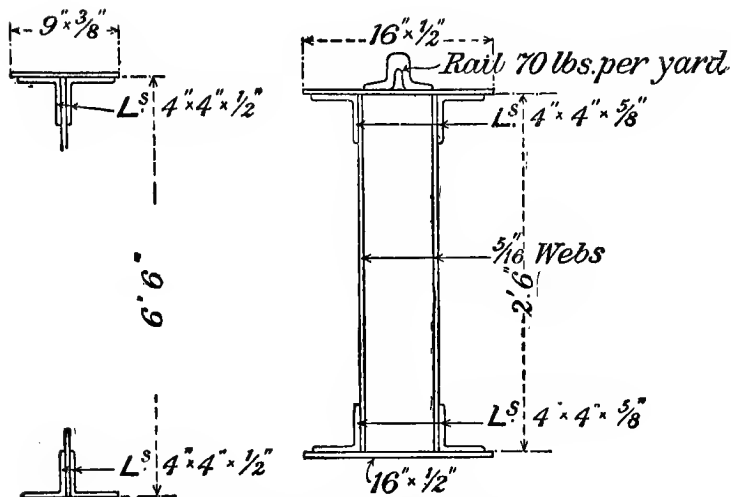


FIG. 46.—DETAIL DIMENSIONED VIEWS OF ROOF AND CRANE GIRDERS.

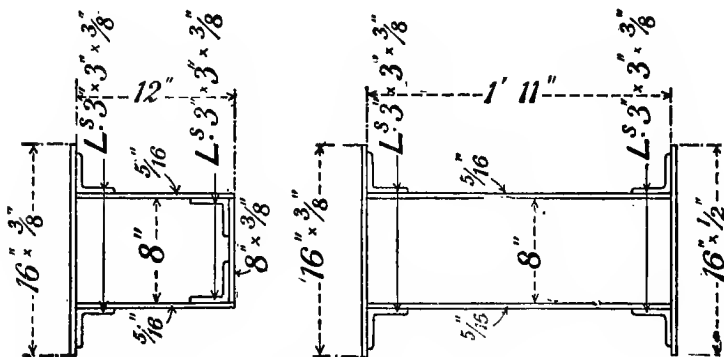


FIG. 47.—DETAIL DIMENSIONED VIEWS OF THE UPPER AND LOWER PARTS OF COLUMNS A A.

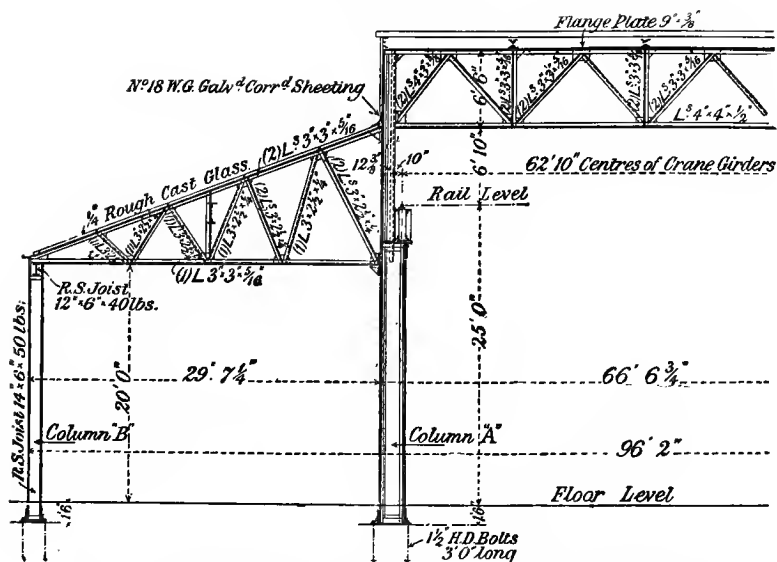
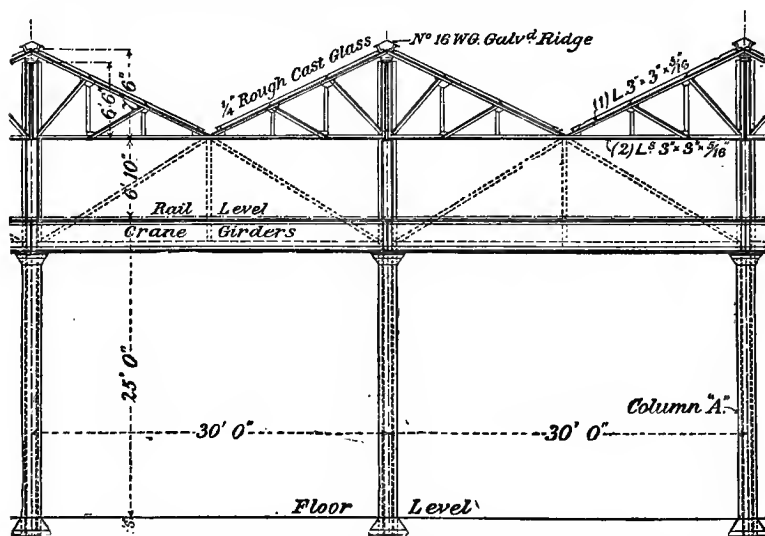


FIG. 48.—CROSS-SECTION OF BUILDING, BY SIR WM. ARROL & CO.

FIG. 49.—SECTIONAL ELEVATION OF BUILDING ON COLUMNS A A,
(Extension of Sir Wm. Arrol & Co.'s Glasgow Works.)

LECTURE III.—QUESTIONS.

1. Explain and prove the rule for obtaining the resultant of the stresses in all the bars of a frame crossing any given section. In Fig. 37*a* what is the resultant of the stresses in the bars A P, P K, and also in the bars A M, M N, and N F?

2. The dimensions of an iron king post truss for a roof are :—Span, 20 feet; height, 7 feet; distance between trusses, 8 feet. The roof weighs 12 lbs. per square foot. Find the stresses in each part.

3. In the above question find the stresses when the wind causes a pressure of 30 lbs. per square foot on one slope.

4. Explain the differences caused in the stresses by the different methods of completing a queen post frame. Mention some of the advantages and disadvantages of each.

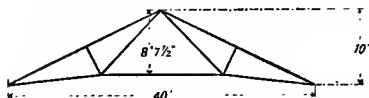
5. A queen post roof has a span of 30 feet, and is 10 feet high. The roof weighs 10 lbs. per square foot, and the principals are 10 feet apart. Find the several stresses if the rafters and tie-beam are continuous.

6. If there is a wind pressure of 25 lbs. per square foot on the roof in Question 5, find the stresses in the bars.

N.B.—See *Appendices B and C* for other questions and answers.

LECTURE III.—A.M. INST. C.E. EXAM. QUESTIONS.

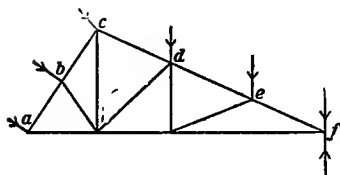
1. Give a reciprocal diagram of the stresses in the bars of the roof shown in the sketch, loaded with 2 tons at each joint of the rafters.



(I.C.E., Oct., 1897.)

2. Show how to find, graphically or otherwise, the loads borne by the supports of a roof when the weights and wind forces are specified. Give an example.

- (I.C.E., Oct., 1897.)
3. Draw a diagram of stresses for the roof frame shown in the fig., when carrying a vertical load of 2,000 lbs. at each of the joints *b*, *c*, *d*, and *e*, and 1,000 lbs. at *a* and *f*, and a uniformly distributed normal wind-pressure on the left side 6,000 lbs.—assuming all the joints to be flexible. Show for each bar whether the stress is push or pull. (I.C.E., Feb., 1898.)



4. In the "queen" truss used for iron roofs of large span, the straight rafters and horizontal tie-rod are connected by a triangulation of equidistant vertical suspending rods and diagonal struts; find the stress on each member of the truss when the load on the roof is uniformly distributed. (I.C.E., Oct., 1899.)

5. Sketch the common king-post truss employed in roofs of small span, and explain how the stress on each member is calculated, so far as is due to roofing material of given weight per square foot. (I.C.E., Feb., 1900.)

6. Describe the action of wind-pressure on a roof. In the last question, assuming the upward reaction of one of the walls on which the roof rests to be vertical, find the stress on each member due to wind-pressure of known amount. (I.C.E., Feb., 1900.)

7. In a bowstring bridge the platform is suspended by vertical suspending rods, without diagonal bracing, from a pair of parabolic arched ribs which may be assumed to be jointed at crown and springing. Describe the straining actions produced in the ribs by a load at the centre of the bridge and calculate their maximum values. Explain the object of the diagonal bracing introduced in practice. (I.C.E., Feb., 1900.)

8. A load of 1200 lbs. hangs suspended at a point, *B*, by a pair of ropes, *BA* and *BC*, to the two points of attachment, *A* and *C*, on the underside of a sloping rafter. The ropes, *BA* and *BC*, are each 16 feet long, and the rope, *BA*, is horizontal, while the point, *C*, is 8 feet higher (measured vertically) than the points, *A* and *B*. Find the direct pull in each of the ropes, *BA* and *BC*. (I.C.E., Oct., 1900.)

9. Show with the help of an example how you would proceed to determine the stresses in the members of a simple roof-truss, produced by a given horizontal wind-pressure, and also the reactions on the points of support (the horizontal reaction is supplied by the right-hand wall).

(I.C.E., Oct., 1901.)

10. If you were supplied, for a given roof-truss, with a table of the maximum stresses due to the dead load, and also those due to the wind-pressure, how would you proceed to select the working stresses per square inch which it would be safe to allow in each bar? Explain fully the reasons for any method you adopt. (I.C.E., Oct., 1901.)

11. Show with the help of an example how you would proceed to determine the stresses in the members of a simple roof-truss, produced by a given horizontal wind-pressure, and also the reactions on the points of support; the horizontal reaction is supplied by the right-hand wall. (I.C.E., Oct., 1901.)

N.B.—See *Appendices B and C* for other questions and answers.

LECTURE IV.

CONTENTS.—Wharf Crane—Example I.—Common Jib Crane—Balanced Jib Crane—Derrick or Scotch Crane—Foundry Crane—Shear Legs—Example II.—130-ton Steam Crane—Tables of Dimensions and Weights of 130-ton Crane—Examples III. and IV.—Questions.

Wharf Crane.—Suppose, as in Fig. 1a, that a single movable pulley carries the load W . Then, neglecting friction, the pull throughout the chain will be one half of W . Again, assume that the pull of the chain acts at the centre of the pulley or barrel round which it may be passing. In the Frame Diagram, Fig. 1a, the external forces acting on the frame are all duly indicated. At the Jib end, there are two forces—viz., the pull of gravity $D E$, on the supported mass, acting vertically downwards and the force $C D$, due to the pull in the chain which is assumed to be parallel to the tie-rod $C H$. At the top end of the vertical post, there are the forces $B C$ and $A B$ acting as shown, which are both due to the pull in the chain on the pulley at the post head. There is also a force acting at the centre of the barrel along the crane

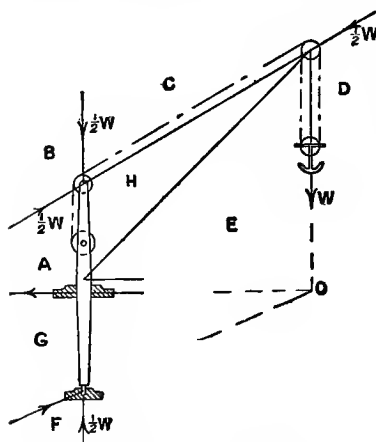


FIG. 1a.—FRAME DIAGRAM.

WHARF CRANE

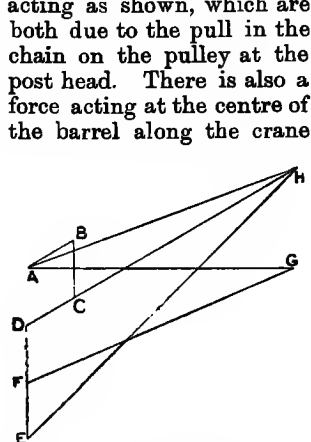


FIG. 1b.—STRESS DIAGRAM.

post. This force must be transferred to the footstep as shown and is called $E F$ in the diagram. There is also a pressure transmitted by the sole plate to the vertical post. This pressure is

assumed to have its line of action horizontal and is lettered $G A$. Finally, we have the action of the footstep on the lower end of the vertical post.

The three forces $D E$, $F G$ and $G A$, must form a system in equilibrium. Therefore, since the lines of action of $D E$ and $G A$ are known, if we produce them to meet in O , then the line of action of $F G$ is known because it must also pass through O .

The Stress Diagram, Fig. 1*b*, may now be drawn. Draw the line of loads $A B$, $B C$, $C D$, $D E$, and $E F$. Now, draw $F G$ parallel to the line of action of the force $F G$ and $A G$ parallel to the line of action of the force $A G$. These close the External Force Polygon. Then if $C H$ and $E H$ be drawn parallel to the bars $C H$ and $E H$ respectively they fix the point H . The line joining the point A with the point H is the finishing line of the Stress Diagram. This line is not parallel to the bar $A H$ because the bar $A H$ is subject to bending.

In Fig. 2, we have a representation of the forces acting on the vertical post; from which, we can determine the bending, tension, and compression stresses in the crane post.

The lengths of the lines $C H$ and $E H$ in the Stress Diagram, Fig. 1*b*, give the stresses in the tie-rod and jib respectively. The horizontal component of $G F$ gives the shear on the bolts of the footstep and $G A$ the shear on the bolts of the sole plate.

EXAMPLE I.—In a wharf crane the post, tie-rod, and jib measure 15, 20, and 30 feet respectively, what would be the nature and amount of the stresses in each of the three members when a load of 7 tons is suspended over the pulley at the jib head, (1) when the lifting chain passes from the pulley to the drum or barrel parallel with the jib, (2) when the drum is placed so that the chain passes from the jib head parallel with the tie-rod? (S. and A. Exam., 1890.)

ANSWER.—First, draw to scale a Frame Diagram $A B C$, as shown. This will be coincident with the centre lines of the different members of the crane.

Case (1).—Here the lifting chain passes from the pulley at the jib head parallel to the jib, and, neglecting the friction of the pulley, we shall have two equal external forces at the joint O due to the tension in the two parts of the chain.

In order to draw the Stress Diagram, we may first proceed to

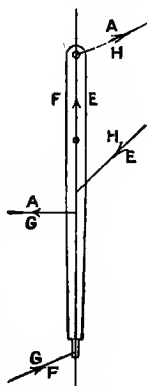
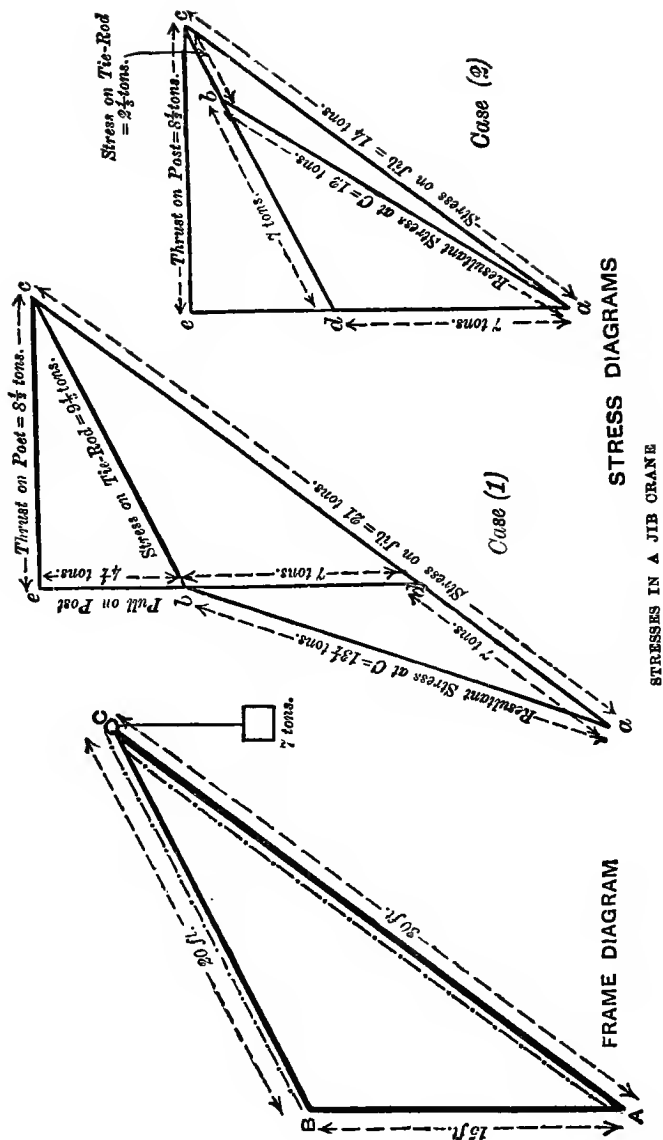


FIG. 2.—FORCES ACTING ON VERTICAL POST OF CRANE.



determine the resultant of these two forces and consider it as a single external force applied to the joint C, and then draw the triangle of forces. Or we may at once draw the polygon of forces for the joint. Thus, draw bd to represent the load of 7 tons, and da equal to bd , and parallel to OA , to represent the tension in that part of the chain over the pulley; then drawing ac and bc respectively parallel to AC and BC , we determine the point c , and therefore the magnitude of the stresses in the jib and tie-rods. These will evidently be compression and tension respectively. If we join ba we obtain the resultant external force at the jib head, and bac will be the triangle of forces determining the same stresses as above.

The nature and amount of the stress in the post will depend on the mode of fixing it. It is evident that the pull bc in the tie-rods may be resolved into a vertical component be , producing tension in the post, while the horizontal component ec represents the force tending to bend the post round A .

Case (2).—Here the chain passes from the pulley parallel to the tie-rods. We proceed as before, and draw bd to represent the pull in the part of the chain above the pulley, and da the pull in the vertical part of it, then ac and bc drawn parallel to AC and BC respectively, determine the point c , and represent the stresses in the jib and tie-rod. Again joining ba , we see that this represents the resultant external force at the pulley. The remainder of the diagram is the same as in Case (1).

The magnitudes of the different stresses are shown on the diagrams, and enable us to compare the relative merits of the two arrangements. Thus, if we suppose the load to be suspended from the end of the jib without the intervention of a pulley, we get bdc in Case (1), or dac in Case (2) as the corresponding Stress Diagram. The effect of introducing the chain and pulley is in Case (1) to *increase* the thrust in the jib by 7 tons—i.e., the pull in the chain—without affecting the pull in the tie-rods, while in Case (2) the effect is to *diminish* the pull in the tie-rods by the same amount—7 tons—without increasing the thrust in the jib. Thus, other things being equal, Case (2) is the better arrangement.

Common Jib Crane.—In the common Jib Crane represented in Fig. 3, the movable pulley has one sheave, and the chain passes direct to the barrel from the Jib-head. The barrel is carried by the cast-iron framing. There are two tie-rods inclined at an angle θ degrees to the centre line of the crane as shown by the plan of the tie-rods. We assume the cast-iron frame to be freely jointed where the tie-rods and the jib meet it, and also where the horizontal part meets the upright post.

The forces acting on the frame of the crane are indicated in Fig. 4a. At the Jib-head there are the two forces W and $\frac{1}{2} W$. At the point in the bar GH , representing the centre of the barrel, there is a force $\frac{1}{2} W$, indicated by the dotted line and arrow head. The lines of action of the dotted force and the

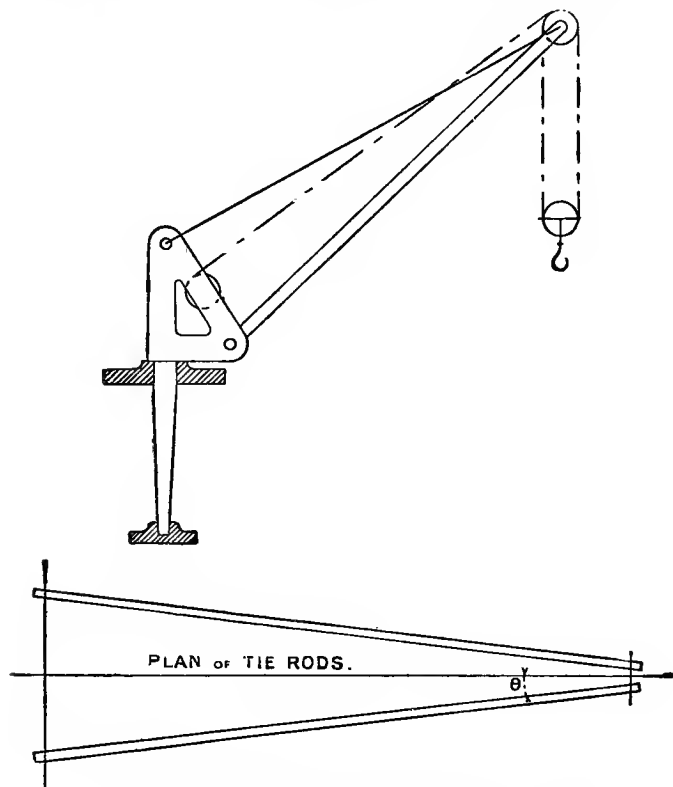


FIG. 3.*—OUTLINE DIAGRAM OF COMMON JIB CRANE.

force BC are coincident. They lie along the line joining the centre of the Jib-head pulley and the chain barrel.

We have here an example of a force acting at a point in a bar. The force acting at a point in the bar GH as represented

* The Plan of the Tie-Rods of this crane has been drawn to a larger scale than the crane itself.

into two components, one along each tie-rod. This is done in the Stress Diagram by drawing from H and from B lines HM and BM each making an angle with BH of θ degrees. Then the lengths of HM and BM measure to scale the stresses in the tie-rods.

Balanced Jib Crane.—The balance weight BW acting at G, is usually mounted on rollers in order that it may be moved nearer to the central post A when the load W is reduced. In this way the moments of the load and balance weight may be kept in equilibrium and thus prevent any undue bending action on the

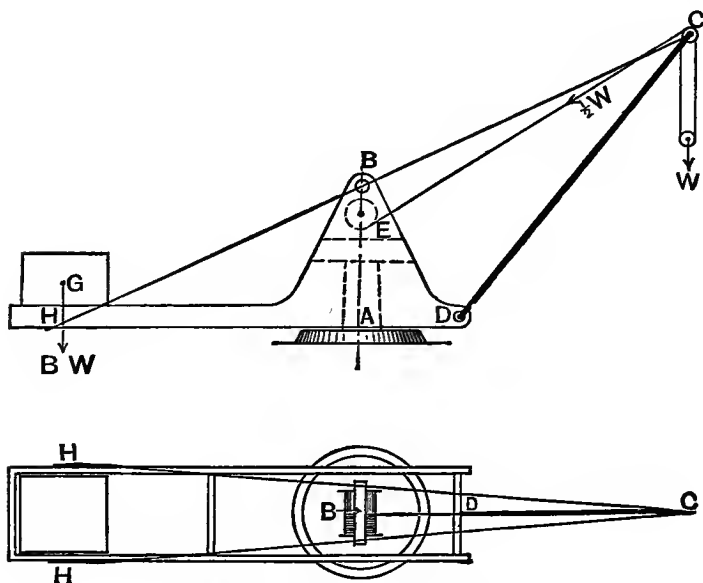


FIG. 5.—BALANCED JIB CRANE.

post at A. We may here remark that the balance weight BW at G and the load W at C are not necessarily equal.

A single movable pulley carries the load W and therefore the tension throughout the chain is $\frac{1}{2}W$. We assume that bars join B with D; D with A; A with B; and A with H. These are all indicated in the Frame Diagram of Fig. 5. The line joining the centre of the pulley C with the centre of the chain barrel E is considered as the line of action of the stress in the chain. From the plan it will be seen that there are two tie-rods

inclined to each other; these rods are often made continuous from C to H.

The load of $\frac{1}{2}W$ acting on the bar FG, Fig. 6a, is divided as explained in the previous example into forces BC and KA. If W is known, we can find on completing the Stress Diagram, the magnitude of the balance weight AB required to balance the moment of the load W about the joint FGEKA. Or, if the balance weight be moved nearer the crane post we can find what weight may be placed in the crane hook.

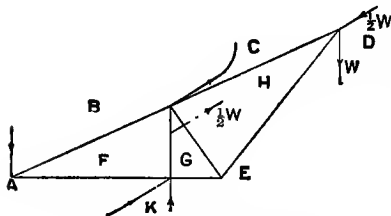


FIG. 6a.—FRAME DIAGRAM.

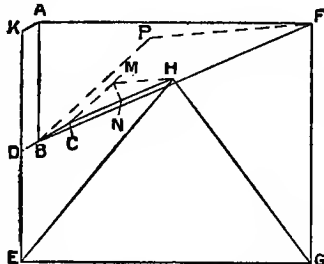


FIG. 6b.—STRESS DIAGRAM.

BALANCED JIB CRANE.

We begin the Stress Diagram, Fig 6b, with the line BC, then CD, and DE. We next find the point H, then, G, F, A, and finally K.

The actual stress in the tie-rods is found, by drawing HM and CM at an angle with HC, equal to half of the real angle between the tie-rods, as previously explained. From F and B draw FP and BP inclined to FB at half the angle between the parts of the tie-rods carrying the balance weight. If the tie-rods are continuous, then HM and FP are parallel.

Derrick or Scotch Crane.*—In Fig. 7, AB is the central upright post, capable of turning round A and B. BC is the jib and AC the tie-rod, which is usually a chain for raising or lowering the jib. The vertical post AB is kept upright by the back stays AE and AE₁. These stays are sometimes anchored to the ground, but are generally attached to the bars BE and BE₁. Boards are placed over these bars and stones or pig iron are placed thereon to act as counterweight to the load W. This crane is similar to that in Fig. 5, in

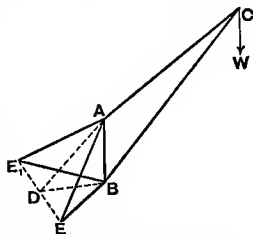


FIG. 7.—DERRICK CRANE.

* See figure at end of this Lecture.

that back balance weights are used. The weights in Fig. 7 are, however, not required to be made movable in order to produce a moment round B equal and opposite to the moment of W round the same point. This is due to the fact, that the under sides of the bars B E and B E₁ rest upon the ground, and, therefore, no matter how much the moments of the balance weights round B may exceed the moment of the load, there is no bending stress produced on the pivot at B.

The chain carrying the load W is usually parallel to the tie-rod A C. It then passes round a pulley at A and down to the barrel on the upright post.

By reference to the Stress Diagrams, Figs. 1b and 6b, the Stress Diagram for Fig. 7 may be drawn. When the plane of the triangle A B C in Fig. 7, coincides with the plane of the triangle A B E, the stress in A E will be a maximum, the stress in A E₁ will be theoretically zero, and the weight required at E may then be found. Similar considerations will give the stress in A E₁ and the weight required at E₁.

Let the plane of the triangle A B C now occupy any intermediate position between the planes of the triangles A B E and A B E₁. Then the stresses in A E and A E₁, may be found by producing the plane of the triangle A B C to intersect the planes A E₁ E and E₁ B E in A D and B D. Now proceed to find the stress in A D as if D were anchored to the ground, then resolve this stress along the stays A E and A E₁, as explained for the inclined tie-rods of the two previous examples. The angle E₁ B E is usually a right angle.

Foundry Crane.—The Frame Diagram, Fig. 8a, illustrates the arrangement of the parts of this type of crane. The pulley carrying the load W is attached to a small bogie running between two parallel horizontal beams. The external forces acting on the crane are F G, G A, A B, C D, E F (each equal to W), B C, and D E. The external force A B balances E F, and C D balances G A, therefore the remaining external forces B C, D E, and F G form a system in equilibrium. Their lines of action will pass through one point O, and their magnitudes may be determined by the triangle of forces. These results may be made use of in drawing the Stress Diagram, Fig. 8b. Commence by drawing E F, F G, G A, and A B all equal in magnitude to W, and parallel to their respective lines of action. If we draw B C parallel to the line of action of the force B C, so as to intersect the line through E parallel to the line of action of the forces C D and D E in the point C, then by marking off C D equal to W we complete the external force polygon.

The stress in A H will be unaffected by raising or lowering

either the footstep or the bearing at the top, so long as the line of action of the load and the position of the joints 1, 2, and 3 remain unchanged. Let us suppose that the footstep coincides

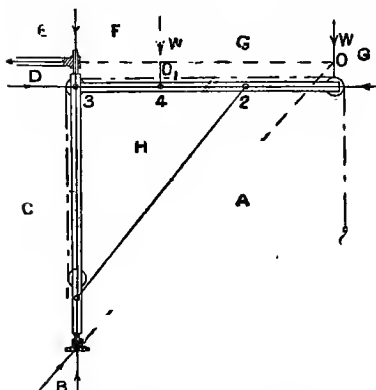


FIG. 8a.—FRAME DIAGRAM.

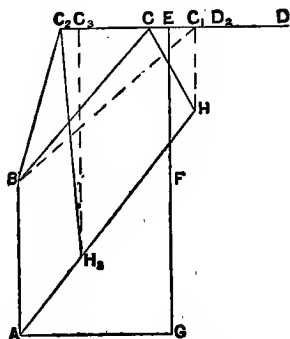


FIG. 8b.—STRESS DIAGRAM.

FOUNDRY CRANE.

with the point 1, and the bearing at the top with the point 3, then the line of action of the stress in CH will be along the

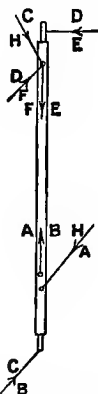


FIG. 9.—FORCES ACTING ON UPRIGHT OF FOUNDRY CRANE.

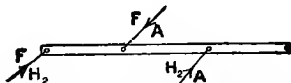


FIG. 10.—FORCES ACTING ON JIB WITH PULLEY AT POINT 4 ON FIG. 8a.



FIG. 11.—FORCES ACTING ON JIB IN FIG. 8a WITH PULLEY AT END OF SAME.

centre line of the bar CH, while the line of action of the footstep reaction BC_1 will pass through the point 1 and the centre of the pulley carrying the load.

In the Stress Diagram, Fig. 8*b*, draw BC_1 through the point B parallel to the line joining the point 1 with the centre of the pulley carrying the load, so as to cut the line CD in the point C_1 ; then by drawing C_1H and AH parallel to the bars CH and AH respectively, we determine the point H. Finally, C joined with H finishes the Stress Diagram. When the pulley carrying the load occupies the position represented by the point 4, then C_2 , D_2 , and H_2 (found in a similar manner) are the corresponding points in the Stress Diagram.

Fig. 9 represents the forces acting on the upright, which

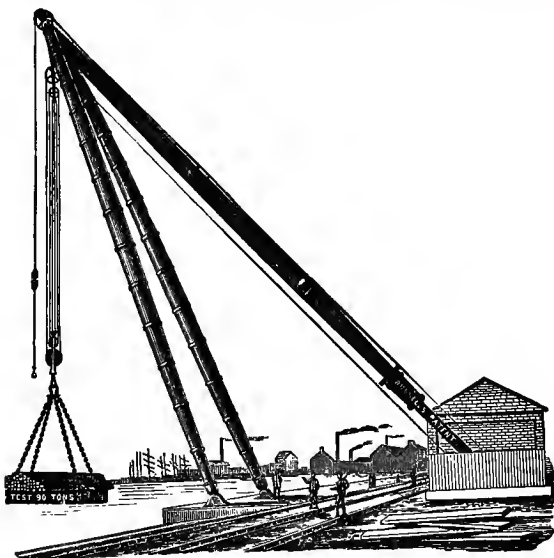


FIG. 12.—RUSSELL'S PATENT SHEER LEGS.

produce bending, tension, and compressive stresses, for the case when the movable pulley is at the end of the horizontal jib.

In Figs. 10 and 11, if the maximum bending moment in each case be the same, then the point 2 in Fig. 8*a* has been so chosen, as to make the bending moment on the horizontal jib have its least value whilst the pulley carrying the load passes from one end of the jib to the other. (See Ques. 41 of Hons. S. & A. Exam. in Machine Constn., 1890.)

Sheer Legs.—A common appliance for lifting engines and boilers into ships is the sheer legs or sheers. The illustration

shows one that has been erected at West Hartlepool by Messrs. George Russell & Co., of Motherwell, for a load of 80 tons overhanging 38 feet 6 inches. It consists of two tubular front legs, each 105 feet long, swinging upon pins at their lower ends, and connected together at the top, which is supported by a hollow stay or back leg. This stay is fixed to the gunmetal nut of a forged steel screw, which rotates inside the back leg. The screw is anchored at its lower end, and can be rotated by a hydraulic engine. As the screw revolves one way or the other, the back leg is shortened or lengthened, and the top is moved in or out, as shown on Fig. 13. The total horizontal travel thus given to the load is 50 feet.

Chains worked by a pair of hydraulic engines are used for lifting, and there are separate chains for light and heavy loads. The latter chain operates a six-purchase pulley block.

In Fig. 13, $A_1 B$ is the line of the front legs hinged at B , and $A_1 C$ that of the back leg. The top can move between A_1 and A_2 by altering the length of $A C$. The vertical $A_1 D$ represents the load on the sheers (80 tons), and $A_1 E$ the tension in the chain,

($\frac{1}{6}$ of 80 or $13\frac{1}{3}$ tons since there are six chains supporting the load). Draw $E F$ parallel to $A_1 D$, and $D F$ parallel to $A_1 E$. Then $A_1 F$ is the resultant force to be balanced by the stresses in the legs $A_1 B$, $A_1 C$. Draw $F G$ parallel to $A_1 C$, and meeting $A_1 B$ produced, if necessary, in G . Then $A_1 G$ (160 tons) is the compression transmitted to the front legs, and $F G$ (76 tons) the tension in the back leg. As the two front legs are not parallel, we must, in order to determine the actual stress in each, draw a second figure, as shown at the right-hand side. Here $H M$ is equal to half $A_1 G$, and $K H M$ and $L H M$ are each half the

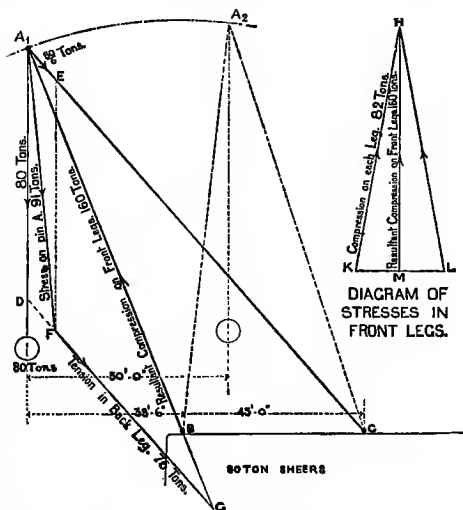
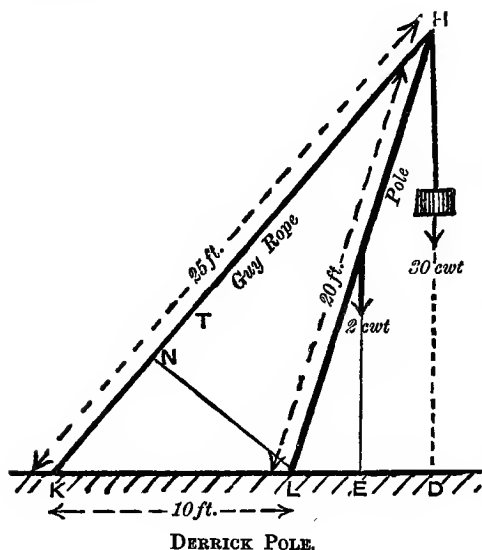


FIG. 13.—COMBINATION OF FRAME AND STRESS DIAGRAMS FOR FIG. 12.

angle between the front legs. KL being drawn perpendicular to HM , we obtain HK and HL (82 tons) as the compression in each leg.

EXAMPLE II.—The foot of a uniform derrick pole of weight 2 cwts. rests on the ground, and the pole carries a weight of $1\frac{1}{2}$ tons suspended from its upper extremity. The length of the pole is 20 feet, and it is kept in position by a guy rope fastened to the ground 10 feet to the rear of the foot of the pole and 25 feet in length. Find by construction or otherwise the tension of the guy rope.



ANSWER. — In the figure, LH is the derrick and KH the guy rope. The weight of the derrick acts at its centre, and may be replaced by 1 cwt. at each end, so that the total load in the upper end may be taken as 31 cwts. This is the force AB on the Frame Diagram. For the Stress Diagram draw AB parallel to the force AB and equal to 31 units. Make BC parallel to the pole and AC to the guy

rope. Then, AC is the tension in the guy rope and is equal to 25 cwts. and BC ($= 53$ cwts.), the compression in the pole.

This problem could have been solved by the Principle of Moments, as follows:—

Let T be the Tension in the guy rope.

„ w „ Weight of the pole.

„ W „ Weight hung from pole.

Drawing LN perpendicular to KH and taking moments about L (in the first figure), we have:—

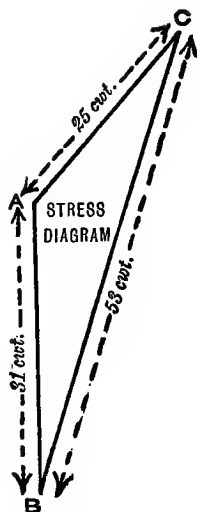
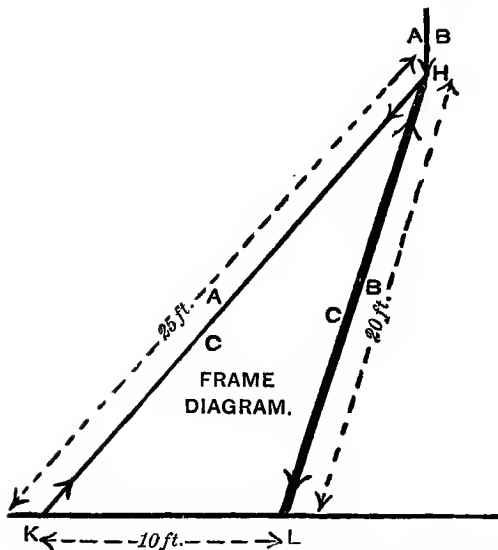
$$T \times L N = W \times L D + w \times L E$$

$$\text{Or,} \quad T = W \times \frac{L D}{L N} + w \times \frac{L E}{L N} = (W + \frac{1}{2} w) \frac{L D}{L N}.$$

From Euclid II., 12, we have :—

$$K H^2 = K L^2 + L H^2 + 2 K L \cdot L D$$

$$\therefore L D = \frac{K H^2 - K L^2 - L H^2}{2 K L} = \frac{625 - 100 - 400}{20} = 6.25 \text{ ft.}$$



STRESSES IN A DERRICK POLE.

$$\text{Also, } L N \times K H = \text{twice triangle } L K H = H D \times K L$$

$$\text{Or, } L N \times K H = (\sqrt{L H^2 - L D^2}) \times K L$$

$$\therefore L N = \frac{K L}{K H} \sqrt{L H^2 - L D^2} = \frac{10}{25} \sqrt{400 - 39}$$

$$\text{Or, } L N = \frac{2}{5} \sqrt{361} = \frac{38}{5} = 7.6 \text{ ft.}$$

$$\text{Hence, } T = (30 + 1) \frac{6.25}{7.6} = \frac{31 \times 6.25}{7.6} = 25.5 \text{ cwts.}$$

130-Ton Steam Crane.*—As an example of a very large crane we have illustrated, in the plate appearing on the opposite page to this, the 130-ton steam crane erected for the Clyde Trustees at Finnieston Quay, Glasgow, by Messrs. Cowans, Sheldon, & Co., Limited, of Carlisle. A similar crane has also been put up at the new Cessnock Dock, Glasgow. The jib of this crane is made up of two steel tubular girders braced together by diagonal and cross stays. The tension rods have been sawn out of solid steel plates, and were not heated during their manufacture. They are connected to the jib by stays at intervals along their length. The foot of the jib is attached to one of the bottom corners of a large vertical triangular frame, and the tension rods to the upper corner, while the back one supports the balance weight which is placed between the two sides of the main framing. The boilers and engines are also placed within this framing, which is covered in so as to form an engine-house. The whole is fixed on the top of a circular base, which can rotate around a large central pin, and rests on steel rollers running on a steel pathway on the top of the foundation. There is also a roller bearing between the base and top of the centre-pin. The foundation, which is square in plan, is of concrete with granite corners and cope, and is supported on twenty-two concrete cylinders sunk into the sand. The centre piece of the crane is fixed to the foundation by six steel bolts cottered to washer plates in a tunnel inside the foundation.

There are two separate lifting blocks, the one for heavy, and the other for light weights. Each of these can be raised or lowered at two different speeds. Separate engines are provided for each of these blocks, and for rotating the crane. All three sets of engines have two cylinders with cranks at right angles, so as to start from any position. Steel wire ropes are used for hoisting instead of chains. The heavy weights are taken on an eight-purchase pulley block, and the light weights on a double-purchase pulley block. All the gearing, up to 24 inches diameter, is of cast steel, and the remainder of a mixture of cast-iron and steel. Gun-metal bushes are used throughout.

The crane is provided with a 160-ton Duckham hydrostatic weighing machine, and was tested by loading it with 150 tons of steel rails. Its radius of action is 65 feet, and the total lift is 100 feet.

The following tables show some of the leading dimensions and weights :—

* For a complete description of this crane see *Engineering*, June 9, 1893.

PARTICULARS OF 130-TON CRANE AT FINNIESTON QUAY, GLASGOW.

Part.	Number.	Length.	Breadth.	Diameter.	Thickness or Height.	Height above or below Quay.	Total Weight in Tons.	Projects beyond Cope of Quay Wall.
Foundation above front cylinders, Foundation, Concrete cylinders,	22	40' 0" 36' 6"	40' 0"	9' 7½" and 5' 9½" Centre 10' 9"	45' 0"	Top 20' up	4,300	5' 6"
Tunnel and Passage,	...	...	2' 0"	...	6' 0"	Top 10' down	...	...
Foundation bolts,	6	38' 9" 6' 0"	6' 0"	...	...	10' down.	8	...
Washer Plates,	6	...	...	...	...	...	13	...
Centre Piece for Crane,	...	...	...	...	...	...	9	...
Centre Pin,	...	...	...	0' 17"	...	...	6	...
Roller Path,	...	...	...	33' 0" 0' 14"	...	...	12	...
Rollers,	75	...	...	...	...	...	10-5	...
Framing,	...	...	...	6' 0"	27' 0"	...	50	...
Boiler,	...	...	...	...	14' 0"	...	6	...
Back Balance,	...	...	...	...	...	...	100	...
Jib,	...	90' 0"	...	Tubes 3' 3" at centre to 2' 6" at ends.	1½" at cen- tre to 1" at ends.	Top 110' up.	45	...
Single Tension Rods,	...	...	0' 10"	...	0' 2½"	...	15	...
Double	...	...	0' 10"	...	0' 1½"	...	...	...
Pins for	...	...	...	0' 8"	...	...	...	44' 3"
Pulley for light weights,	...	...	...	2' 6"	...	107' 6"	...	39' 6"
Pulley for heavy weights,	...	...	...	5' 3"	...	100' 0"	...	44' 3"
Gin block for light weights,	...	...	...	...	...	...	...	39' 6"
Gin block for heavy weights,	4 pulleys	12' 0"	7' 0"	Pulleys 5' 3" 2' 6" 5' 2"	3' 0"	...	7	...
Hoisting Drum for light weights,	...	...	...	...	...	...	...	...
Hoisting Drum for heavy weights,	...	10' 0"	...	...	...	...	10-5	...
Gearing,	...	...	...	...	...	...	8	...
All Castings,	...	...	...	...	...	...	120	...
Crane in working order, exclusive of back balance,	...	...	...	...	...	...	270	...
Radius of sweep for light loads,	...	69' 9"	...	...	...	...	...	...
Radius of sweep for heavy loads,	...	65' 0"	...	...	...	...	...	...

PARTICULARS OF ENGINES FOR 130-TON CRANE.

For Load.	Engines.			Distance Raised in One Minute.	Time to make Complete Revolution of Crane.
	No. of Cylinders.	Diameter.	Stroke.		
130 tons, . . .	2	12"	18"	4'	5 mins.
60 " . . .				8' 10½"	2 mins. 17 secs.
20 " . . .	2	8"	12"	28'	...
8 " . . .				60'	...
Revolving, . . .	2	8"	12"	...	...

PARTICULARS OF STEEL-WIRE ROPES FOR 130-TON CRANE.

Rope.	Inner Core.		Second Layer.		Third Layer.		Outer Layer.		Total No. of Wires in each Strand.	No. of Strands.	Total No. of Wires.	Remarks.
	No. of Wires.	Diam. of Wires.	No. of Wires.	Diam. of Wires.	No. of Wires.	Diam. of Wires.	No. of Wires.	Diam. of Wires.				
Heavy Lifts, .	3	·078"	9	·084"	15	·084"	18	·101"	45	7	315	Six strands are of patent steel, and are wound round a central strand of soft steel.
Light Lifts, .	1	·060"	8	·080"	12	·060"	18	·060"	37	7	259	

The factors of safety allowed in the different parts are :—

Main framing, jib, tension rods, &c., ...	6
Wire ropes, ...	8
Centre holding down bolts, to allow for rusting, .	12

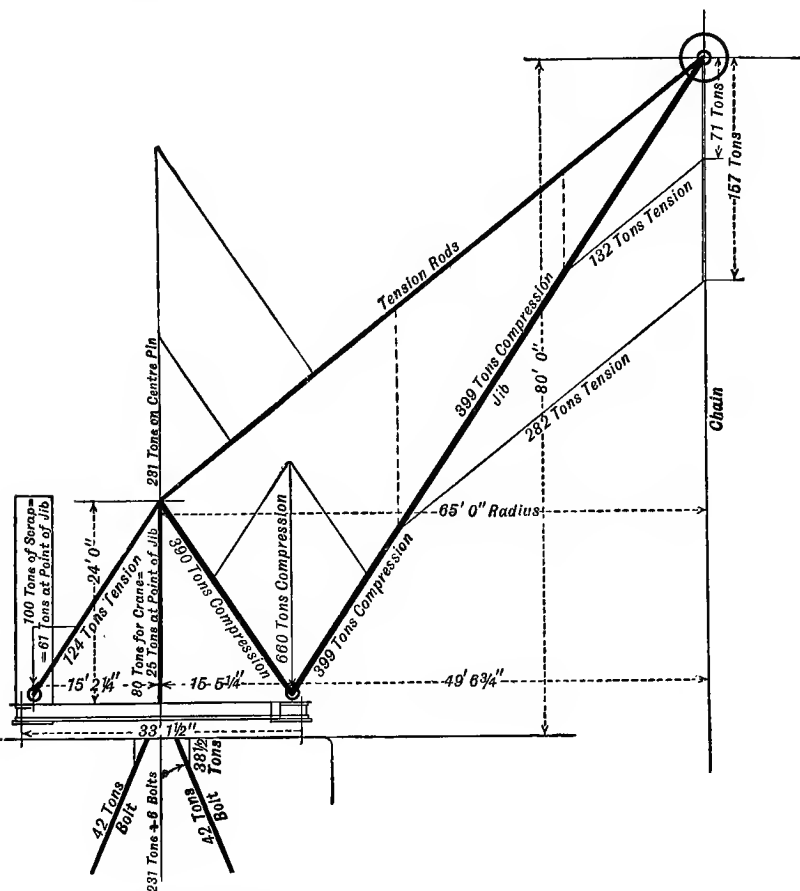


FIG. 14.—STRESS DIAGRAM FOR 130-TON CRANE. (TEST LOAD, 150 TONS.)

The accompanying figures show the Stress Diagram for the crane when loaded with 150 tons, or, including the weight of the gin block, 157 tons in all, and also for a gross load of 71 tons.

Fig. 14 is the Stress Diagram as worked out by the makers. Fig. 15a represents the Frame Diagram and Fig. 15b the Stress Diagram worked out for the Test Load as we have treated the previous cranes—viz., all in one figure.

First, find the values of the Forces D E and A B in Fig. 15a as already explained. Then begin the Stress Diagram with the force A B and follow with the forces B O, C D, D E, E F and F G. Now E L and G L fix the point L; B H and C H the point H; H K and L K the point K; K N and G N the point N; and N M and A M the point M.

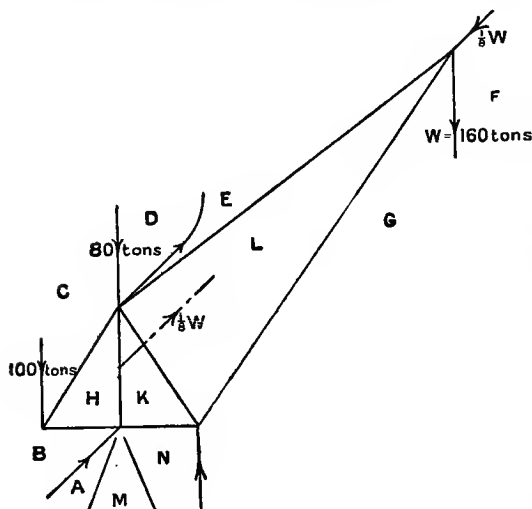


FIG. 15a.—FRAME DIAGRAM.

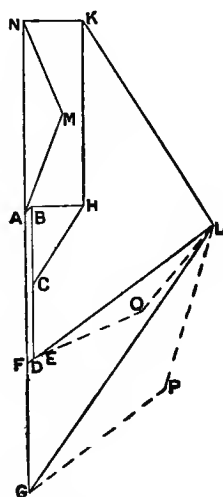


FIG. 15b.—STRESS DIAGRAM.

130-TON CRANE WORKED OUT AS IN THE PREVIOUS EXAMPLES.

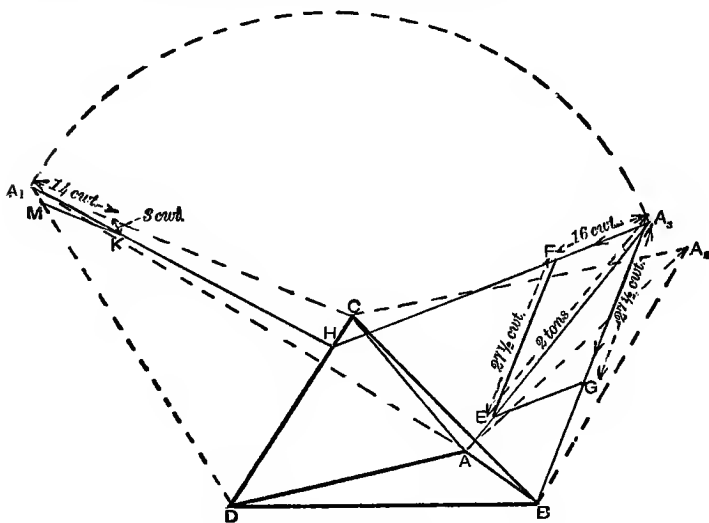
The stress E L is borne by the two tie-rods and L O and E O represent the stresses in each tie-rod. The stress G L is divided over the two jibs, consequently L P and G P represent the stresses in each jib, as already explained.

The stress A M is divided over half of the holding down bolts and N M over the other half. The point about which the crane tends to topple over is the joint K L G N.

EXAMPLE III.—A tripod whose vertex is A, and whose legs are A B, A C, A D, of lengths 8, 9, and 10 feet respectively, sustains a load of 2 tons. The ends B, C, D form a triangle, whose sides are B C = 7 feet, C D = 6 feet, B D = 8 feet, find

by graphical construction the compressive stress in each leg. (S. and A. Exam., 1889.)

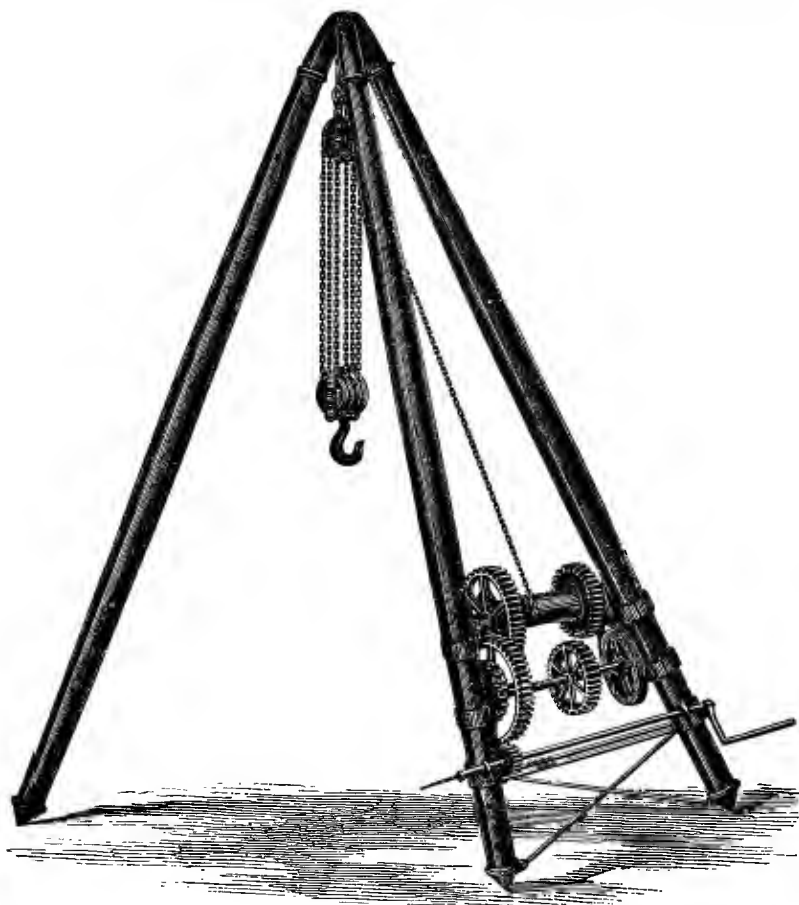
ANSWER.—Hitherto we have dealt only with forces in one plane, or symmetrical with respect to one plane. Thus, in the case of the shear legs, in determining the compression on the front legs, we first found what would be the compression on an intermediate leg in the same plane as the two replaced, and equally inclined to both. This hypothetical leg would be in a vertical plane containing the back leg and the externally applied force, and evidently the stress in the back leg would not be affected by the substitution. In the present example we must find the



STRESSES IN A TRIPOD.

corresponding intermediate leg, so that if the load be supported by it and the third leg, the stress in the latter will not be altered. It will be in a vertical plane containing the third leg, and will be represented by the line of intersection of this plane with that of the other two legs, hence the following solution:—Draw BCD the triangle formed by the feet of the tripod; to find the plan of the vertex, draw the triangle DCA₁, making CA₁ and DA₁ equal to CA and DA respectively. Similarly, draw the triangle A₂CB; then A₁A drawn perpendicular to CD, and A₂A drawn perpendicular to CB, will give by their

intersection the plan of the vertex A , and we may now complete the plan. The vertical plane containing the leg AB will cut the line DC in a point H lying on the line BA produced;



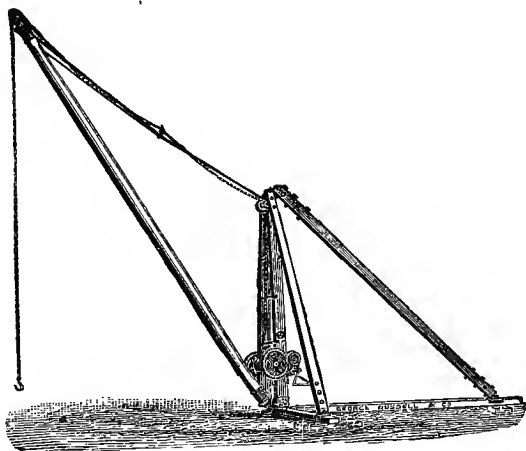
TRIPOD FOR LAYING PIPES.

then A_1H will be the length of the required intermediate leg. Draw A_3A_2 at right angles to BAH , and make $HA_3 = HA_1$. Thus we get A_3H , A_3B , and the applied force A_3E in one

plane. The stress in the leg AB will be represented by A_3G , and the resultant stress in the legs AC and AD —*i.e.*, the stress in the hypothetical leg AH by AF . Dividing this between the actual legs by the triangle of forces A_1MK , where $A_1K = AF$, we get the stress in AC represented by MK , and that in AD by A_1M . The respective values are marked on the diagram.

As a practical example of the use of a tripod, we illustrate a form much used—in laying water and other mains—for lowering large pipes into position. Two of the legs are braced together, and carry a winch which may be used in conjunction with a block and tackle. The pipe is rolled on to wooden beams laid across the drain, and the tripod then placed in position over it. The pipe is slightly raised by means of the winch, and the wooden beams removed, when the pipe may be lowered with ease.

We also illustrate a simple hand crane as used by contractors for building, &c. It is of the kind discussed in connection with Fig. 7.



HAND DERRICK CRANE.

EXAMPLE IV.—Calculate the main dimensions for a set of four folding tubular shear legs to fulfil the following conditions:—(1) To stand 15 feet high when the feet are set on a circle of 3·5 feet radius; (2) to safely sustain a load of 4 tons; (3) the load to be lifted by means of a differential pulley block; (4) the tops of the legs are to be connected by a mild steel pin which passes through the eyes of all four and also through the sling hook; (5) the legs are to be made of ordinary welded wrought-iron steam-pipe $\frac{1}{2}$ inch thick, and are not to be braced in any way. (*From Technics.*)

ANSWER.—If we assume that the load is such as may require to be drawn to one side after a vertical lift, it is possible that, nearly all the load will be supported by one leg. Consequently, we can, therefore, take it that, the whole load may come upon one leg.

Since the legs are 15 feet high, when standing on a circle of 3·5 feet radius, the true length of each leg will be $\sqrt{15^2 + 3\cdot5^2} = 15\cdot4$ feet. Hence, if we draw a triangle, to any scale, with a perpendicular to represent 15 feet; and a base and an hypotenuse to represent 3·5 and 15·4 feet respectively, then it will serve to give the *slope* of the legs, as well as a *triangle of forces*.

Should the load of 4 tons be dragged sideways, so that the chain sustaining the load lies practically parallel to the shear leg, which we are supposing to take the whole load, then the magnitude of compression can be found by proportion. Thus, referring to the triangle of forces—

$$\frac{\text{Height}}{\text{Hypotenuse}} = \frac{15 \text{ feet}}{15\cdot4 \text{ feet}} = \frac{4 \text{ tons}}{x \text{ tons}}$$

$$\therefore x = \frac{4 \times 15\cdot4}{15} = 4\cdot1 \text{ tons.}$$

Now, as the whole load cannot really fall entirely on one leg of the shear, the true load will be somewhat less than 4·1 tons; say, 4 tons.

As this is our original figure, we are thus led to the simple working rule of designing each leg as a strut capable of safely bearing a longitudinal pressure equal to the total load to be carried.

Since each leg will be acting as a strut with hinged or rounded ends, the load per square inch will require to be considerably less than the safe compressive stress on a cube or short length of the given material, so that, this strut can only be designed by a method of trial and error.

Take Gordon's formulæ for a wrought-iron strut with rounded ends, i.e.—

$$\text{Crippling load} = \frac{16 \text{ tons} \times \text{area of strut.}}{1 + \frac{4}{2,000} \left(\frac{\text{length}}{\text{diameter}} \right)^2}.$$



FOUR FOLDING TUBULAR
SHEAR LEGS.

$$\text{Working load} = \frac{\text{crippling load}}{\text{factor of safety}}.$$

$$\text{Factor of safety} = 4 + \left(\frac{\text{length}}{\text{diameter}} \times \frac{1}{20} \right).$$

If we try a strut **6 inches in diameter** we will find that, the working load will be about 4·5 tons, therefore this size of ordinary welded wrought-iron steam-pipe will be satisfactory.

In order to find the diameter of the pin, we may assume that the two intermediate legs carry the load, and that the other two legs serve chiefly for steadying the shear legs and load. Of course this assumption does not at all represent the true conditions to be met with in actual practice, but gives stresses not less than the actual. Also, the mechanical and structural conditions may necessitate the centres of the intermediate eyes of the shear legs being placed about 6 inches apart.

Therefore, treating the pin as a beam with a central load, the bending moment will be—

$$B M = \frac{W l}{4} = \frac{4 \text{ tons} \times 6 \text{ inches}}{4} = 6 \text{ ton-inches.}$$

The modulus of resistance to bending = $R M$ = modulus of section $\times$ maximum safe bending stress.

The modulus of section of a round bar of diameter $D = \frac{D^3}{10}$ approximate.

Take the maximum safe working stress at 7 tons per square inch.

Then, assuming a diameter of 2 inches for the pin, we get—

$$R M = \frac{D^3}{10} \times 7 = \frac{2 \times 2 \times 2}{10} \times 7 = 5\cdot6 \text{ ton-inches.}$$

As the pin may bear on the inner edges of the eyes, the effective span may be less than 6 inches, so that a pin 2 inches in diameter will be sufficiently strong.

If the pin is strong enough to resist bending it will be quite strong enough to resist the shearing forces. The diameter of the pin given here may appear excessive, but there is a tendency to design these hinge pins as though they were merely to resist double shear under ideal conditions, forgetting that, with folding shears the pin is subject to much ill-usage in setting up and taking down the shears.

The above methods are only to be adopted for shears worked by hand, because for the larger sizes worked by mechanical means there are many intricate conditions involved.

LECTURE IV.—QUESTIONS.

1. In a model of a crane the jib is $3\frac{1}{2}$ feet long, the tie-rod is 3 feet long and is fastened to a point 1 foot vertically above the lower end of the jib. What is the thrust on the jib when a weight of 20 lbs. is hung at the upper end of it? *Ans.* 70 lbs.

2. In a wharf crane the post, tie-rod, and jib measure 15, 20, and 30 feet respectively, what would be the nature and amount of the stresses in each of the three members when a load of 7 tons is suspended over the pulley at the jib head, (1) when the lifting chain passes from the pulley to the drum or barrel parallel with the jib, (2) when the drum is placed so that the chain passes from the jib head parallel with the tie-rod?

3. In a hydraulic wharf crane the height of the post is 6 feet, the jib is 22 feet, and the tie-rod is 18 feet; find the horizontal thrust on the post when 5 tons are supported. In what way is the friction which opposes the *slewing* motion reduced to a minimum? *Ans.* 12·24 tons.

4. Find, either graphically or otherwise, the stresses on the jib and tie-bar respectively of a crane, whose jib measures 20 feet in length, when the tie-bar and post are 16 feet and 6 feet in length respectively, and a weight of 25 cwts. is suspended from the end of the jib. The line of direction of the chain after leaving the barrel or drum runs parallel to the tie-bar. Also calculate the pressure on the end of the handle of 16 inches radius when the weight is lifted, supposing the drum of the crane to be 15 inches in diameter, and the gearing to consist of a pinion of 12 teeth, gearing into a wheel with 72 teeth, while a second pinion of 18 teeth gears with a wheel of 56 teeth.

5. A contractor's portable hand crane has a vertical post AB, to which the jib AC is inclined at 45° , and the tension rod BC makes with AB an angle ABC of 120° . The back stay from the head of the post B to the extremity D of the horizontal strut AD is inclined at an angle of 45° to AD. Find the weight of the counterbalance required at D to balance a load of 10 tons suspended from the end C of the jib. Determine also the nature and amount of the stress on the jib AC, and in the rods BC and BD? (The tension in the chain may be neglected.) (S. and A. Adv. Exam., 1896.)

6. A jib foundry crane consists of a vertical post AB, 16 feet long, fitted with pins working in sockets at both A and B. From the upper end A of the post extends a horizontal member AD, 28 feet in length, and from the foot B is a strut BC, which meets AD at a point 16 feet from A. A load of 20 tons being suspended from D, find the shearing stress on the pin at A, and the stress along the strut BC.

7. A sheer-legs is formed of two sheer-poles BC, DC, each 25 feet in length, and secured to a base-plate in the ground at B and D. The wire guy or tension rope AC is attached to the ground at a point A, which is 60 feet distant from BD. The perpendicular from the top C of the poles meets the ground at a distance of 10 feet from the centre of the line BD, which is 15 feet long. Find by measurement or otherwise the tension in tons on the guy when a weight of 20 tons is suspended. *Ans.* 11·3 tons.

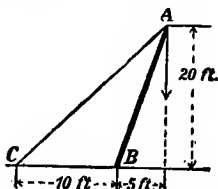
8. The jib of a 10-ton crane is inclined at 45° to the vertical, and the tie rod at an angle of 60° . Find the thrust on the jib and the pull on the rod, the tension in the chain being neglected. If the tension in the chain is by means of tackle, half the load and the chain barrel is so placed that the direction of the chain bisects the crane post, find the forces in the jib and tie-rod. Graphical constructions may be used. (C. & G., 1902, H., Sec. A.)

N.B.—See Appendices B and C for other questions and answers.

LECTURE IV.—A.M. INST. C.E. EXAM. QUESTIONS.

1. A pair of shear legs make an angle of 20° with one another and their plane is inclined at 60° to the horizontal. The back-stay is inclined at 30° to the plane of the legs. Find the force on each leg, and on the stay, per ton of load carried. (I.C.E., Oct., 1897.)

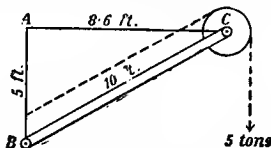
2. A set of shear legs consists of 2 struts, A B set 10 feet apart at the



lower ends (B), and a stay rod AC. Determine graphically the stresses in the struts and in the tie when a load of 20 tons hangs from A.

(I.C.E., Feb., 1898.)

3. A weight of 5 tons hangs upon the chain of the bracket or crane here sketched. The jib BC has an inclination of 30° with the horizontal tie-rod AC; so that if BC is 10 feet, AB is 5 feet, and AC is 8.6 feet. At



C the chain is carried over a frictionless pulley, and led down parallel to the jib. Find the stresses in the members AC and BC.

(I.C.E., Feb., 1899.)

4. A load is suspended from a pair of shear legs supported by a back leg, the foot of which is moved by a horizontal screw. Show by diagrams the stress on each leg when the load overhangs by a given amount, and find the resistance overcome by the screw in moving the legs.

(I.C.E., Oct., 1899.)

5. In a portable crane stability is obtained by a counterweight placed at the end of a horizontal piece stayed to the crane-post in the usual way. Show by diagrams the stress on each part of the structure, and the magnitude of the counterweight when a load is simply suspended from the end of the jib. (I.C.E., Feb., 1900.)

6. Explain how you would determine graphically whether five given coplanar forces acting at a point are in equilibrium or not. In a portable erecting-shop crane the height of the crane post is 8 feet; angles of back stay and jib with crane post, each 45° ; angle of front stay with crane post, 120° . Find, by graphical means, the nature and magnitude of the forces in the various parts, and determine the necessary amount of the balance weight applied at the extremity of the back stay when 5 cwts. are being lifted. (I.C.E., Oct., 1900.)

7. In a wall-crane the jib is 10 feet long, and makes an angle of 45° with the wall. Determine graphically the position of the tie-rod, so that the tension in it may be a minimum for a given load on the crane. Then, assuming the tie-rod to be fixed in this position, determine graphically (a) the stresses in the tie and jib, when the load of 5 tons is suspended from a chain, which passes over a pulley at the junction of the tie and jib to some point on the wall, so that it bisects the angle between the tie and the jib. (b) How these stresses are affected by the position of the chain relative to the tie and jib, taking as extreme positions, (i.) when the chain is parallel to the tie, (ii.) when the chain is parallel to the jib. (The diameter of the pulley may be neglected.) (I.C.E., Oct., 1901.)

8. A weight of 10 tons is suspended from a tripod of equal legs whose feet rest on the ground in an equilateral triangle of side 12 feet, and whose apex is 25 feet from the ground. Calculate the stress in each leg.

(I.C.E., Oct., 1902.)

9. A crane has a vertical crane-post, AB, 8 feet long, and a horizontal tie, BC, 6 feet long, AC being the jib. It turns in bearings at A and B, and the chain supporting the load passes over pulleys at C and A, and is then led away at 30° to AB. Find the stresses in the bars and thrusts on the bearings when lifting 1 ton at a uniform rate. (I.C.E., Feb., 1903.)

N.B.—See Appendices B and C for other questions and answers.

LECTURE V.

CONTENTS.—Reactions on a Beam—First Method—Resultant of the Loads on a Beam—Reactions on a Beam—Second Method—Fink Truss—Trapezoidal Truss of Three Panels—Trapezoidal Truss of Five Panels—Example I.—Warren Girder—Linville or N Girder—Lattice Girder—Redundant Frame—Five Bay Lattice Girder—Lattice Girder loaded at Top Joints—Bending Moment—Definition—Shearing Force—Definition—Cantilever Uniformly Loaded—Examples II. and III.—Centre of Gravity of an Area—Moment of Inertia of an Area—Proof—Engine Mechanism—Questions.

Reactions on a Beam.—FIRST METHOD.—First Case.—In the Frame Diagram, Fig. 1*a*, we have a beam with five concentrated loads upon it. The line of action and point of application of the left-hand reaction, and the point of application of the right-hand reaction are known.

We commence Fig. 1*b* by drawing the line of loads, B C, C D, D E, E F and F G. Then, any point O is taken and joined with all the points in the line of loads as illustrated in the figure. This point O is called a Pole and Fig. 1*b* a Polar Diagram.

Since the point of application of the right-hand reaction (Fig. 1*a*) is the only one of its elements which is known, we must begin at this point 1 when drawing the Funicular Polygon* 1 2 3 4 5. In the Polar Diagram, Fig. 1*b*, the line O F comes between the loads E F and F G. Then, from the point 1, in the Frame Diagram, draw the line 1—2, parallel to O F. The line 1—2 begins in the line of action of the load F G and ends in the line of action of the load E F—viz., the two loads between which the line O F lies in the Polar Diagram. Between the lines of action of the loads E F and D E, draw the line 2—3 parallel to the line O E in the Polar Diagram, which lies between these same two loads. Similarly draw 3—4 parallel to O D and 4—5 parallel to O C. On joining the point 5 with the point 1 we close the Funicular Polygon. This closing line 5—1 has one end 5, in the line of action of the reaction A B, and the other end 1, in the line of action of the reaction A G. Therefore, if a line O A be drawn parallel to this closing line from O, it must lie between the reactions in the line of loads, just as the other lines radiating from O have done.

If we draw B A from B in the line of loads parallel to the line of action of the reaction A B, so as to meet the line O A in the

* See explanation of Funicular Polygon given at the end of this Lecture.

point A, then A joined with G completes the external force polygon.

Second Case.—If the two ends of the beam had been anchored, then the lines of action of the reactions G A and A B would have been parallel to each other and to the line joining B with G in the Load Diagram. In this case the lines of action of the

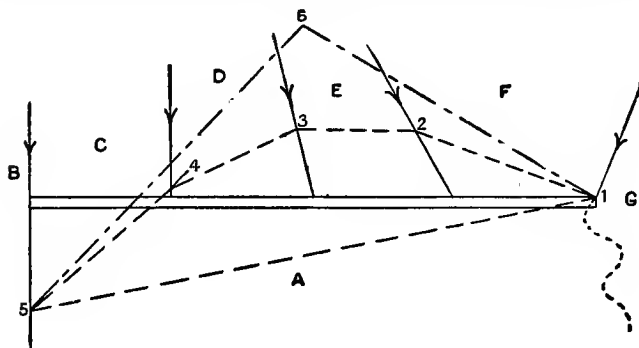


FIG. 1a.—FRAME DIAGRAM AND FUNICULAR POLYGON.

reactions would be drawn on Fig. 1a, and the point 1 taken anywhere in the line of the reaction G A; or we could begin with the point 5 anywhere in the line of the reaction A B, and draw the Funicular Polygon as stated above.

If the load F G had not existed, we would still have begun at the point 1 for the first case. The line O F would then be between the load E F and the reaction F A. In the second case mentioned above, the solution would still be the same as before.

Resultant of the Loads on a Beam.—From the point 1 in the Frame Diagram, Fig. 1a, draw the line 1—6 parallel to the line O G in the Polar Diagram. This line O G lies between the load F G and the resultant G B. Now draw the line 5—6

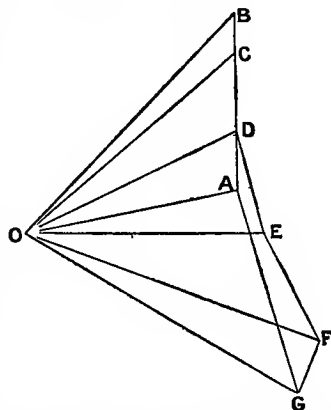


FIG. 1b.—LOAD AND POLAR DIAGRAM FOR DETERMINATION OF REACTIONS AND RESULTANT LOAD.

parallel to the line OB , so as to intersect the line 1—6 in the point 6. Then the point 6 is a point in the line of action of the resultant of the loads. If through the point 6 we draw a line parallel to BG , then that line will represent the line of action of the resultant load. This line cuts the beam at the point round which the beam would balance. The magnitude of the resultant is represented by the length of the line BG .

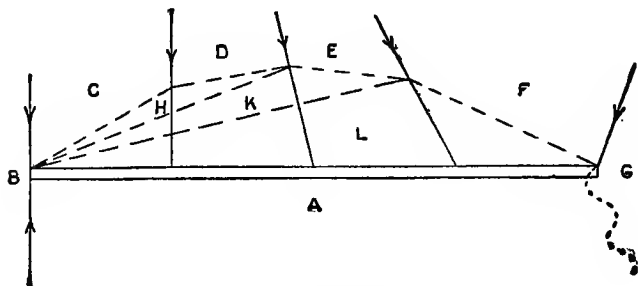


FIG. 2a.—FRAME DIAGRAM.

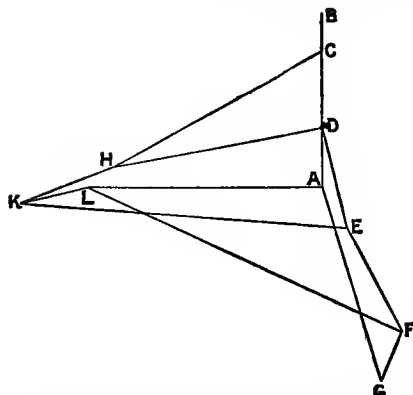


FIG. 2b.—LOAD AND REACTION DIAGRAM FOR DETERMINATION OF REACTIONS.

to the line joining C with F , while its magnitude is represented by the line CF .

Reactions on a Beam.—SECOND METHOD.—On the top of the beam draw a Firm Frame of any shape, having its joints in the lines of action of the loads as indicated in the Frame Diagram, Fig. 2a. Letter the Frame Diagram according to

If we wished to find a point in the line of action of the resultant of the loads CD , DE , and EF , then from the point 2, draw a line parallel to the line OF , and from the point 4 a line parallel to the line OC , so as to intersect the first line from the point 2. This point of intersection is a point in the line of action of the resultant of CD , DE , and EF . Its line of action passes through this point and is parallel

Bow's method, and draw the Load and Stress Diagram as already explained. As a graphic solution the second method is the more accurate of the two, and may be applied with greater ease in the case of lattice girders, for we have only to join the points of the girder with one end, or some points with one end, and the remainder with the other end.

Fink Truss.—This truss was largely used in America for wooden bridges and is represented in the Frame Diagram, Fig. 3a. It consists in this case, of a primary truss B A G, and two secondary trusses. The divisions of the beam are called panels. The truss in Fig. 3a is therefore a Fink Truss of four panels.

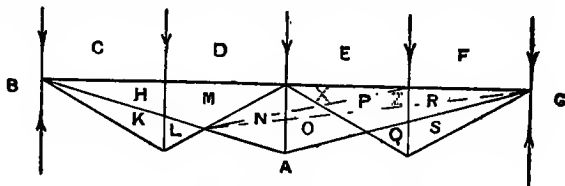


FIG. 3a.—FRAME DIAGRAM, FOR A FINK TRUSS.

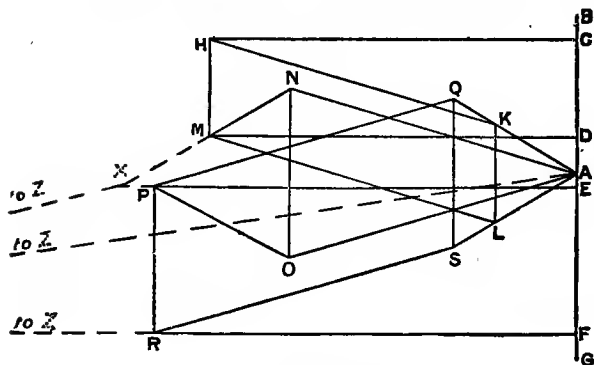


FIG. 3b.—STRESS DIAGRAM.

If the foot of every upright had been joined with each end of the beam instead of as shown, the truss would be called the Bollman Truss, of which the solution is similar to the following explanation for the Fink Truss. Draw the external force polygon as explained for Figs. 1a and 2a.

FIRST METHOD FOR STRESS DIAGRAM.—Join the joint M N A L with the joints E F R P and F G A S R, as shown by the dotted lines in the Frame Diagram, Fig. 3a. Call the spaces thus formed

X and Z as shown in the Frame Diagram. This will enable us to determine the point M in the Stress Diagram, Fig. 3*b*. Draw FZ and AZ, parallel to the bars FZ and AZ. This gives the point Z. Then ZX and EX fix the point X; and XM and DM fix the point M. The Stress Diagram may now be completed in the usual way, and the closing line will form a check line.

SECOND METHOD FOR STRESS DIAGRAM.—In Fig. 4, we have four forces in equilibrium acting at a point and their lines of action lie along two straight lines. The condition for equilibrium is, that HM is equal and opposite to KL and ML is equal and opposite to KH. The Force Polygon will therefore be a parallelogram and may be represented by HMLK in the Stress Diagram, Fig. 3*b*.

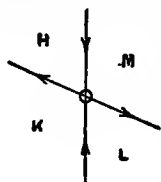


FIG. 4.—FORCES ACTING IN STRAIGHT LINES AT A POINT.

From the above, it is evident that the stress in HM must be equal to the load CD and the stress in KL equal to the stress in HM. Therefore, draw AK and AL parallel to the bars AK and AL respectively and draw KL between them parallel and equal in length to CD. This will determine the points K and L in the Stress Diagram, Fig. 3*b*, which may now be completed. The closing line will form a check as in the first method.

Trapezoidal Truss of Three Panels.—In this Truss, Fig. 5*a*, the space F will always remain a parallelogram, as was the case in the Queen Post Frame which has been already discussed. In fact this is a similar case to the roof considered with a continuous tie-beam.

If the beam is uniformly rigid, then whatever distance the joint CDGF is deflected below the horizontal line, the joint BCFE rises an equal distance above the horizontal line. Therefore, the difference between the vertical component of the load CD and the stress action in GF, must be equal and opposite to the difference between the vertical component of the load BC and the stress action in EF.

In order to obtain the Stress Diagram, Fig. 5*b*, we first of all draw the external force polygon BCDA, and determine the point A, as explained for Figs. 1*b* or 2*b*. Then draw AF parallel to the bar AF and $B_1C_1D_1$ perpendicular to AF. BB_1 , CC_1 and DD_1 are parallel to AF. B_1C_1 and C_1D_1 will be the vertical components of BC and CD respectively.

Since C_1D_1 is greater than B_1C_1 , the joint CDGF will be deflected downwards. Therefore, FG will be less than C_1D_1 .

and is as much less than $B_1 C_1$ as $C_1 D_1$ is less than $E F$. This is the same thing as saying that $G F$ and $F E$ are together equal to $B_1 D_1$.

The point E lies on the line $A E$ drawn parallel to the bar

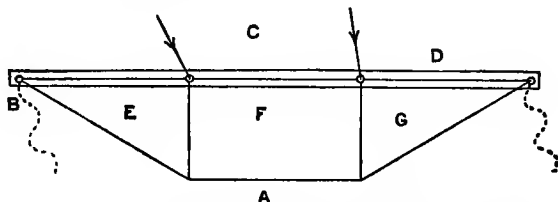


FIG. 5a.—FRAME DIAGRAM FOR A TRAPEZOIDAL TRUSS.

$A E$ and similarly for the point G . Place the line $E G$ between the lines $A E$ and $A G$, parallel to the bars $G F$ and $F E$ and equal in length to $B_1 D_1$. This determines the points E and G . Where $E G$ cuts the line $A F$ fixes the point F . Then by joining $F C$, $E B$ and $G D$ we complete the Stress Diagram.

Another method of determining the point G is to make $A P_1$ equal to one-half of $B_1 D_1$, or $A P$ equal to one-half of $B D$, and draw $P_1 G$ or $P G$ parallel to $A F$, so as to cut the line $A G$. This determines the point G .

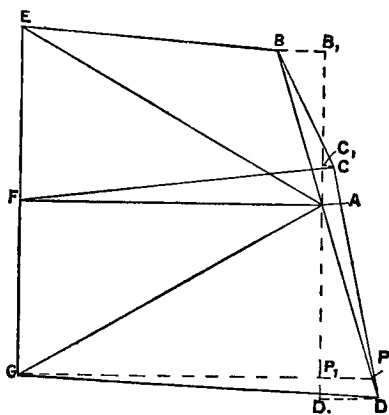


Fig. 5b.—STRESS DIAGRAM.

Trapezoidal Truss of Five Panels.—In this truss, Fig. 6a, the space O always remains a parallelogram, but the other spaces cannot change. Similar reasoning to what we used in the previous figure will show that the loads $D E$ and $E F$ are together equal and opposite to the sum of the stress actions $P O$ and $O N$. The load $O D$ is equal to the stress in $L N$, and also equal to the stress in $K M$. Similarly the load $F G$ is equal to the stresses in the bars $P R$ and $Q S$. It follows from this, that the stresses $R P$, $P O$, $O N$, and $N L$ are together equal and opposite to the loads $C D$, $D E$, $E F$, and $F G$. The Stress

In Fig. 7 the forces acting on the left-hand portion of the beam are shown as taken from the Stress Diagram, Fig. 6*b*. They are the actions of the pins on the beam, and of the right-hand half on that end.

In Fig. 8 is shown the form which the left-hand end must take from the previous assumptions. The three points shown



FIG. 7.—FORCES ACTING ON THE LEFT-HAND HALF OF THE BEAM.

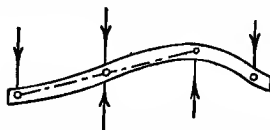
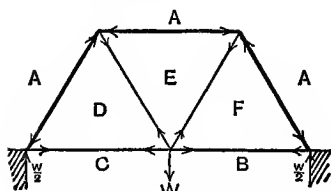


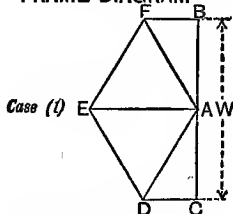
FIG. 8.—FORM INTO WHICH THE FORCES BEND THE LEFT-HAND HALF OF THE BEAM.

in a straight line must always remain in a straight line. There will be three points of contrary flexure in the length of the beam.

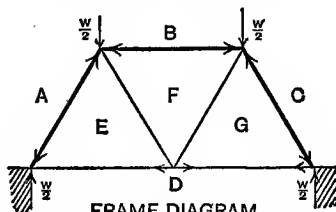
These Trapezoidal Trusses are deficient frames made redundant by stiff joints.



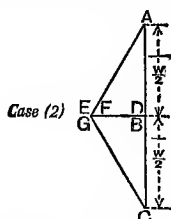
FRAME DIAGRAM



STRESS DIAGRAM



FRAME DIAGRAM



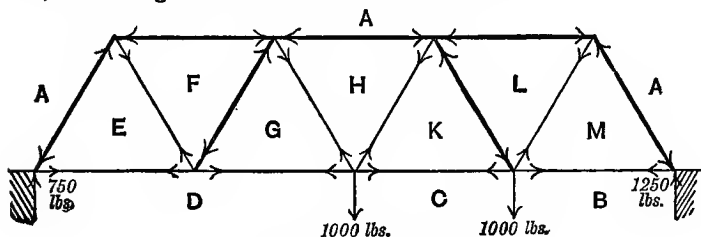
STRESS DIAGRAM

TRIANGULAR FRAME.

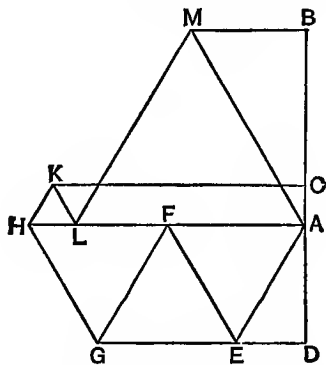
EXAMPLE I.—A triangular frame, consisting of three equilateral triangles, is loaded with a weight W . Find the stresses on the several members of the frame (1) when W is hung at the lower apex of the central triangle, (2) when each of the triangles is loaded at the upper apex with $\frac{W}{2}$.

ANSWER.—Since the loading is symmetrical in both cases, the reactions are each equal to one-half of the total load—that is, to $\frac{1}{2} W$.

Case (1).—Draw BC vertical and equal to W units. Bisect it at A, so that CA and AB are the reactions. Make CD parallel to the bar CD, and AD to AD. This gives us D. Then draw DE and AE respectively parallel to the bars DE and AE, so as to obtain E. AF and EF fix F, and BF completes the Stress Diagram. BF should be parallel to the bar BF, and this gives us a check on our work.



FRAME DIAGRAM



STRESS DIAGRAM

FIG. 9.—WARREN GIRDER.

Case (2).—Here we must make AB and BC each equal to $\frac{1}{2} W$. DA and CD will represent the reactions, D being the middle point of BC, and therefore coincident with B. AE and DE determine E, and CG and DG give G. E and G will coincide since B and D do, and everything is symmetrical. The point F also coincides with E and G, so that there is no stress in the bars EF and FG.

Warren Girder.—Fig. 9 illustrates a Warren Girder of four bays or panels, which is simply an extension of the above triangular frame. The upper horizontal member is called the **Upper Boom or Flange**, and the lower horizontal member is called the **Lower Boom or Flange**. The inclined members are called **Lattice Bars or Braces**. The joint $A F E$ or $E F G D$ is called an **Apex**. The angle of triangulation is usually 60° , but sometimes the triangles are right-angled isosceles. When loads are applied at the centres of the lower members, tie-rods are put from the centres of the lower members to the opposite apices. These tie-rods transmit the central loads to the upper apices. The Frame Diagram would then show a system of loads at the lower and upper apices. From what has already been said the student should find no difficulty in drawing the Stress Diagram, which is obtained in exactly the same way as those in Example I.

Linville or N Girder.—This girder differs from the Warren Girder, in that the bars connecting the two horizontal booms are placed alternately vertical and oblique, forming a series of right-angled triangles instead of the corresponding equilateral ones in the Warren Girder. It is so arranged that the shorter vertical bars shall be in compression and the oblique ones in tension. This clearly tends to a saving of material and a diminution of weight, since compression members, unless very short, must, other things being equal, be much heavier than tension members. Also, compression members are much strengthened by shortening, while a tension member is not weakened by lengthening.

We may determine the reactions by a "substituted frame" by the Funicular Polygon, or, in this case, by calculation.

In drawing the Stress Diagram we observe that $A a$ must be equal to $Q A$, and $a Q$ is zero, since the reaction is vertical. The members $a Q$ and $n K$ are necessary to give the required stability. These, together with the end vertical members, might be dispensed with by carrying the supports up to the upper boom.

Draw the lines representing the loads and reactions, viz.:— $K L, L M, M N, N P, P Q, Q A, A B, B C, C D, D E, E F, F G, G H$, and $H K$. Since $A a = A Q$, and $H n = H K$, so that a coincides with Q , and n with K , we may draw the Stress Diagram as follows:—

$B b$ and $a b$ fix the point b , and $G m$ and $n m$ the point m .										
$P c$	"	$b c$	"	c ,	"	$L l$	"	$m l$	"	l .
$C d$	"	$c d$	"	d ,	"	$F k$	"	$l k$	"	k .
$N e$	"	$d e$	"	e ,	"	$M h$	"	$k h$	"	h .
$D f$	"	$e f$	"	f ,	"	$E g$	"	$h g$	"	g .

By then joining fg we complete the Stress Diagram, and this line should be parallel to the central strut.

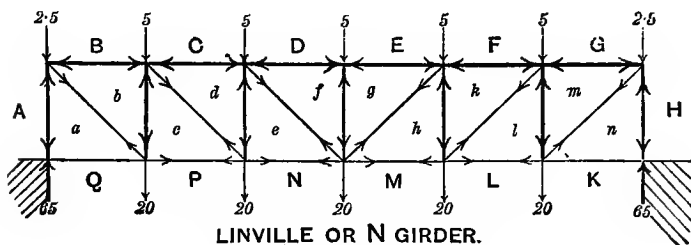


FIG. 10a.

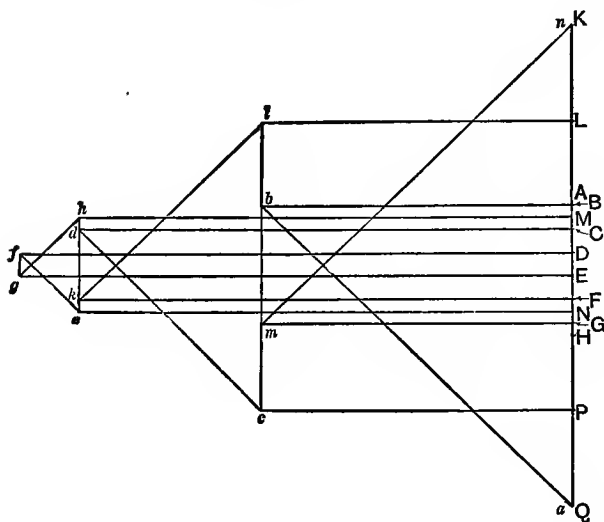


FIG. 10b.—STRESS DIAGRAM.

Lattice Girder.—Although in Fig. 12, we have drawn the external force polygon for Fig. 11, we can go no further with the Stress Diagram for frames of this kind until we have determined in some way the stress in one member, because these are redundant frames.*

* See the beginning of Lecture I. for the properties of redundant frames.

ASSUMPTION.—We assume, first of all, that all the members have been accurately fitted; that is, the frame is not initially stressed.

One method of solution which has often been suggested is, to assume that the shear over any section is taken equally by the braces in that section. We shall however prove that this latter assumption is not consistent with the actual conditions. The shear on the right-hand bay of Fig. 11 is evidently one-half of BC , and according to the above method, the vertical component of the stress in the member GH would be equal to one half of the shear, that is one quarter of BC ,

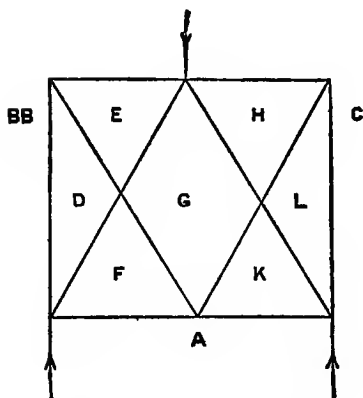


FIG. 11.—FRAME DIAGRAM.

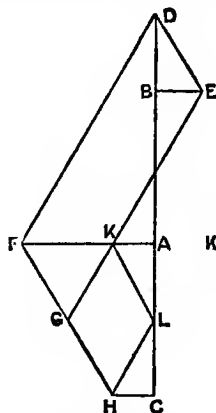


FIG. 12.—STRESS
DIAGRAM WITH
UNEQUAL STRESSES
IN BRACES.

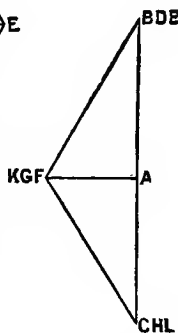


FIG. 13.—STRESS
DIAGRAM WITH
EQUAL STRESSES
IN BRACES.

TWO BAY LATTICE GIRDER.

whilst the vertical component of the stress in the member GK would be equal to the other half of the shear, that is also one quarter of BC . Similarly, for the left-hand bay, according to the above assumption, the vertical components of the stresses in EG and FG would each be one quarter of BC .

Let us suppose the vertical component of the stress in GH and KL , to be one-quarter of BC , and then draw the Stress Diagram, Fig. 12. There AL is equal in length to the vertical component of KL and also equal to one-quarter of BC . The Stress Diagram may now be completed in the usual way. It shows that if the stress in GH has a vertical com-

ponent of one-quarter of BC , then the vertical component of the stress in EG is three-quarters of BC . This is a consistent result, because three-quarters and one-quarter of BC acting vertically together at the joint $E B C H G$ will balance BC . But, if the members have been properly put together, there is no reason at all why the stress in EG should be greater than the stress in GH . The most sensible assumption to make is, that

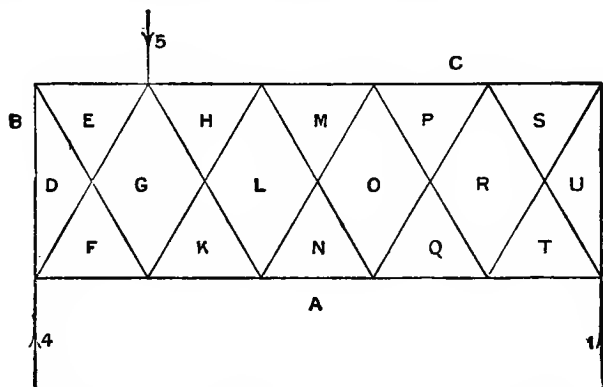


FIG. 14a.—FRAME DIAGRAM FOR FIVE-BAY LATTICE GIRDER.

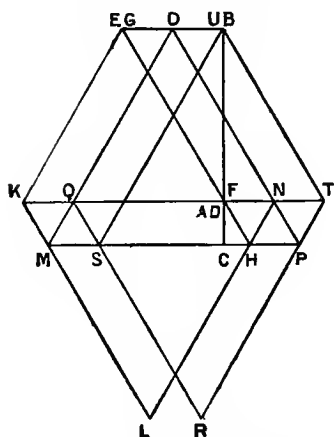


FIG. 14b.—STRESS DIAGRAM.
WHEN BRACES EG AND DF ARE
REMOVED.

the members EG and GH will each have a stress of which the vertical component is one-half of BC . The Stress Diagram for this assumption is worked out in Fig. 13.

Lattice Girder of Five Bays.
—TO DETERMINE WHAT PROPORTION OF A SINGLE LOAD IS TRANSMITTED ALONG EACH BRACE.—In the Frame Diagram, Fig. 14a, we have assumed a load of 5 units at the joint $B C H G E$. From inspection, it will be evident that the reaction AB will be 4 units and AC will be 1 unit. The reactions may be determined by a substituted triangular frame with

its vertex at the loaded joint and rafter ends at the abutments. The Stress Diagram, Fig. 14*b*, has been drawn on the assumption that the braces EG and DF have been withdrawn. If the bar DF is removed, then the spaces D and F will have only one letter, let this be called D. Then BD and AD fix the point D; DF and AF the point F; DE and BE the point E; EG and FG the point G and so on, until the Stress Diagram is completed.

From the Stress Diagram, Fig. 14*b*, we see that the vertical components of the stresses in the braces* GH, KL, LN, MO, OP, QR, RT, SU and UC are each equal to the load BC, and that the vertical components of the stresses in the remaining braces TU, RS, on to BD are each equal to four units; also that the kind of stress alternates between a push and a pull throughout the braces. The vertical component of the push in the lattice bar GH, is balanced by the vertical component of the pull in LN. Also, the vertical component of the pull in MO, is balanced by the vertical component of the push in OP, and similarly for QR and RT, SU and UC. Now, the push in UC is balanced by the reaction CA, combined with the vertical component of the pull in TU, and since the reaction CA is one unit the vertical component of the pull in TU must be four units. This four units of vertical component of the pull in TU, is transmitted through the remaining lattice bars as a push and a pull alternately, until it reaches BD as a push, and is there balanced by the reaction AB.

The stress in each lattice bar produces a strain that will cause the load to dip. Now, since the stresses are severe and every lattice bar is stressed, the removal of the brace EG produces a very yielding frame.

In Fig. 15 we have the Stress Diagram which is obtained from the Frame Diagram, Fig. 14*a*, by removing the brace GH. The removal of this brace means, that there will be no stress in the bars LN, MO, OP, QR, RT and SU. Since there is no stress in SU there can be none in CS or CU. Therefore, S and U will coincide with C and the Stress Diagram can now be completed.

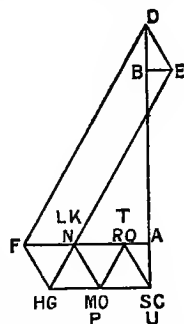


Fig. 15. — STRESS DIAGRAM WITH BRACE GH REMOVED.

* Since the two members GH and KL, Fig. 14*a*, are parts of the same lattice bar or brace and the stresses in them are the same in every element, we may refer to them as the lattice bar or brace GH or KL and similarly for the others.

Fig. 15 shows that the lattice bar EG is the only one having the vertical component of its stress equal to BC. Also that each of the bars BD, DE, GK, LM, OQ and RS has the vertical component of its stress equal to one unit of the load. Further, that the stresses are alternately push and pull and that the remaining lattice bars are not stressed.

The vertical component of the push in the lattice bar EG is balanced by the reaction AB, in combination with the stress in BD. Since the vertical component of the push in the brace EG must balance the load, its value is 5 units and the reaction AB has a value of 4 units; therefore, the stress in the bar BD must be a pull of 1 unit. This vertical component of the pull in BD is balanced by the vertical component of the push stress in DE and so on, until the vertical component of the push in TU is balanced by the reaction AC of one unit.

Since the stress is severe in only one lattice bar and a number of the bars are unstressed, the removal of the brace GH produces a very much less yielding frame than the removal of the brace EG.

When the frame is complete as shown in the Frame Diagram, Fig. 14a, the vertical components of the pushes in the lattice bars EG and GH must together carry the load. The amount which each carries will be inversely proportional to the yieldingness of the system of bars to which each is connected. The lattice bar EG will therefore carry more than the lattice bar GH.

In this case, we have made the following assumptions:—

- (1) That the Frame is not initially stressed.
- (2) That the vertical component of the push stress in the left-hand lattice bar meeting in a joint at which a load is applied is equal to that portion of the left-hand reaction which that load produces. Also, that a similar relation exists between the vertical component of the push stress in the right-hand lattice bar and the right-hand reaction.

Lattice Girder Loaded at Top Joints.—For the lattice girder, Fig. 16a, we must first determine the reactions. This may be done by one of the methods for Figs. 1 and 2, of which the latter one, is the better and simpler. By joining the lower left-hand corner with each of the upper joints we obtain a Simple Triangular Frame, and by drawing the Stress Diagram for it, we can determine the reactions. Since the frame is redundant we must first calculate the stress in one member before we can begin the Stress Diagram. The bar BK is the most suitable one.

From the second assumption we observe that:—

- (1) The load B C, produces no stress in K L and a push stress of 3 units in B K.
- (2) The load C D, produces a push stress having a vertical component equal to 4 units ($\frac{4}{5}$ C D) in K M. This component is entirely balanced by the reaction which the load C D produces in A B—viz., 4 units. This load produces therefore no stress in B K.

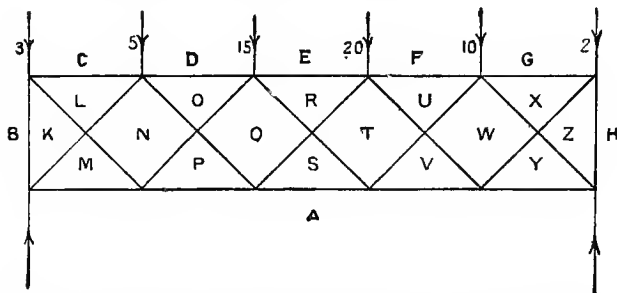


Fig. 16a.—FRAME DIAGRAM FOR A LOADED LATTICE GIRDER.

- (3) The load D E, produces a push stress having a vertical component equal to 9 units ($\frac{3}{5}$ D E) in O Q. This stress induces a pull stress in M N having a vertical component equal to 9 units and this pull stress induces in B K a push stress of 9 units.

- (4) The load E F, produces a push stress having a vertical component equal to 8 units ($\frac{2}{5}$ E F) in R T. This stress induces a pull in P Q,

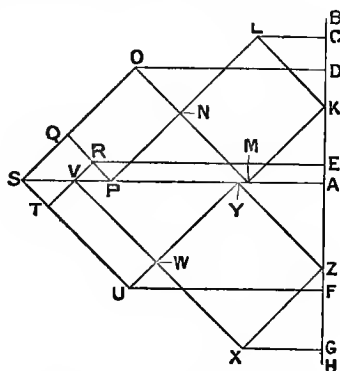


Fig. 16b.—STRESS DIAGRAM.

- and a push in N L, the vertical component of which is balanced by the reaction produced in A B. This load produces therefore no stress in B K.
- (5) The load F G, produces a push stress having a vertical component equal to 2 units ($\frac{1}{5}$ F G) in U W. This stress is transmitted as push and pull until it reaches B K as push and is balanced by the reaction A B.
- (6) The load G H, produces no stress in B K.

Summing up the push stresses in B K we have :—

3 units due to B C,
 0 units due to C D,
 9 units due to D E,
 0 units due to E F,
 2 units due to F G,
 0 units due to G H.

This gives a total push stress of 14 units in the bar B K.

From the point B, in the line of loads, Fig. 16*b*, draw a line B K parallel to the bar B K, and make the length of B K equal to 14 units to the same scale as the line of loads. This determines the point K in the Stress Diagram. Then K L and C L fix the point L and so on point by point until the Stress Diagram is finished. The finishing line forms a check line.

In calculating the stress in the lattice bars, we have sometimes a push stress and sometimes a pull stress. We must therefore pay due regard to the sign of the stress when adding up the various stresses. The force C D produces a push stress in the bar N O, having a vertical component of 1 unit, while E F produces a push stress in R T having a vertical component of 8 units. This push induces a pull in N O also having a vertical component of 8 units. Therefore, the resultant stress in N O is a pull having a vertical component of 7 units.

Referring to Fig. 16*a*, we may observe that sometimes, vertical ties are put in the spaces N, Q, T and W. The action of these ties is to prevent distortion of the rectangles in which they lie. Therefore unless these rectangles have become distorted, they will not be stressed. This distortion will depend upon the relative yieldingness of the two systems of bars forming the rectangle. In this example suppose a tie in the space N and a load applied at the lower joint. Then, before this load can stress the tie, the lower joint must come down more than the upper joint. The safest assumption to make is, that the ties carry no portion of the loads. Some lattice bridges have struts in the spaces N, Q, T and W of Fig. 16*a* and the lattice bars are all ties—that is, they are only able to stand a pull stress. On drawing the Stress Diagram for such a case, we must omit a bar if a push stress comes in it, and use the other tie.

One tie in each bay must carry the shear in that bay. This will enable us to calculate the pull in that tie. This is the best method of drawing the Stress Diagram for such cases.

Bending Moment.—DEFINITION.—The Bending Moment at any point in a beam, is the algebraic sum of the moments with respect to that point, of all the external forces acting on the portion of the beam on either side of that point.

In order to draw the Bending Moment Diagram of Fig. 17, we must proceed as if we were going to find the reactions, by means of the Funicular Polygon and Polar Diagram as explained for Fig. 1a.

The Funicular Polygon drawn in this way is a Bending Moment Diagram.—That is, if a vertical line be drawn from a point in the beam, to cut the bounding lines of the Funicular Polygon,

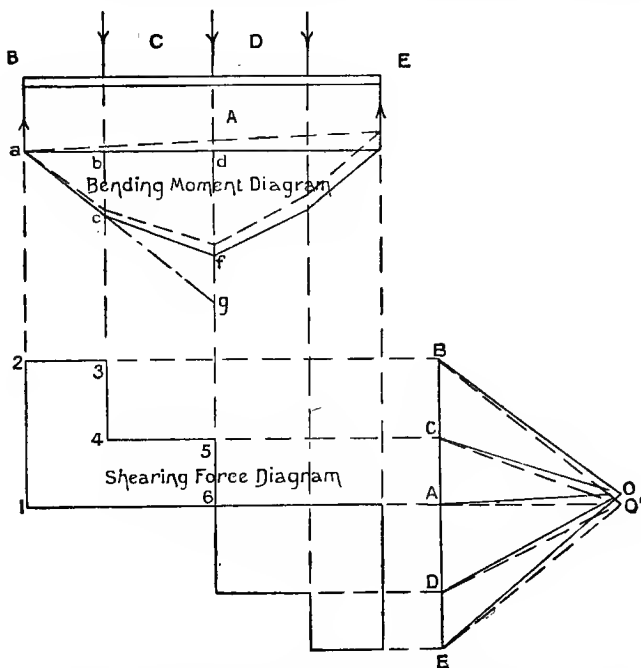


Fig. 17.—BENDING MOMENT AND SHEARING FORCE ON A BEAM.

the intercept on this line which lies between those bounding lines, represents to a certain scale the bending moment on the beam at that point.

Having found the point A in the line of loads, by drawing A O' horizontal and of any suitable length, we draw a Polar Diagram with this point O' as the Pole and the corresponding Funicular Polygon, when we obtain a Bending Moment Diagram on a horizontal base.

PROOF.—The Bending Moment at the point where the load

BC acts, is equal to $AB \times ab$ Units of Moment. The Triangles abc and $O'AB$ are similar, having the sides ab , bc and ca of the one respectively parallel to the sides $O'A$, AB and BO' of the other.

Therefore, $bc : ab :: BA : AO'$

Hence, $bc = \frac{BA \times ab}{AO'}$

Similarly, $df = \frac{BA \times ad - BC \times bd}{AO'}$

That is, the number of units of length in bc , when measured with the scale for the load line, would give the Bending Moment, if $O'A$ measured 1 unit on the scale of length for the Beam.

SCALE FOR BENDING MOMENT DIAGRAM.—Subdivide the unit of the scale used for the line of loads, into as many parts as the line $O'A$ contains the unit of the scale used for the length of the beam. Then, one of these subdivisions will be the unit for the Bending Moment scale. It is found convenient to make $O'A$ ten units of the length scale.

Shearing Force.—**DEFINITION.**—The Shearing Force on any transverse section of a beam is equal to the algebraic sum of all the external forces acting on the portion of the beam on either side of that section.

In order to draw the Shearing Force Diagram of Fig. 17, no explanation is necessary, beyond following out the lines of the figure. The Shearing Force on any transverse section of the beam lying between the loads BC and CD , is, from the definition, equal to the force AB minus the force BC . Therefore, the length between the line 4—5 and the line 1—6 will measure the Shearing Force to the scale of the line of loads.

Cantilever Uniformly Loaded.—The cantilever shown in Fig. 18 may be considered as 12 feet long. The loads indicated are therefore equivalent to a uniform load per foot run. They act at the centre of each of the portions. By drawing from A , B , C , &c., on the Load Line, horizontal lines in the spaces A , B , C , &c., as shown, we determine the Shearing Force Diagram. If we divide the beam into smaller divisions and draw the Shearing Force Diagram, the stepped line will become more nearly a straight line. Consequently, when divided into infinitely small parts, the Shearing Force Diagram becomes the Triangle RPQ . The length of the line QR is the total load on the Beam.

The Bending Moment Diagram of Fig. 18 is determined by drawing in the spaces A, B, C, &c., lines parallel to the lines O A, O B, O C, &c. The limiting form of the curve Q S will be a parabola with its vertex at S, and the value of the length Q T will be the Maximum Bending Moment.

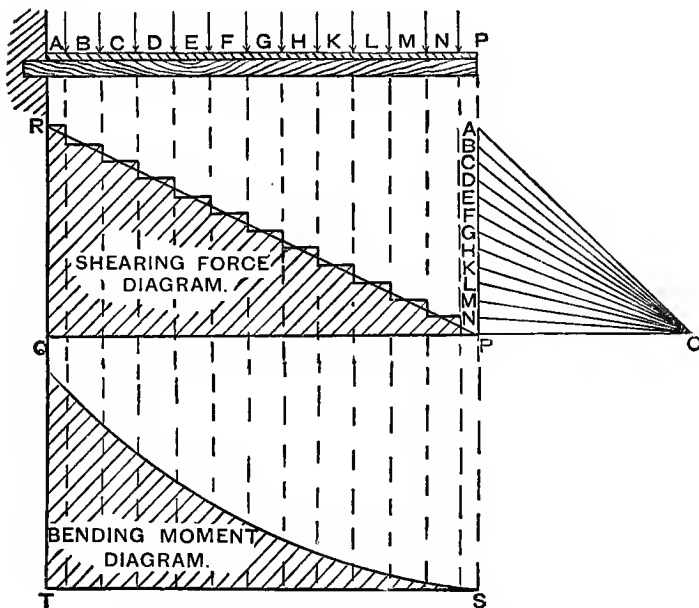


FIG. 18.—UNIFORMLY LOADED CANTILEVER.

Beam Uniformly Loaded and with Concentrated Loads.—Draw, as already explained, the Shearing Force Diagram for the concentrated loads. This is H A 1 2 3, &c., on Fig. 19. Set off H P and K Q, each equal to half the total uniform load on the beam, and join P with Q. Then H P Q K is the Shearing Force Diagram for the Uniform Load. Adding the ordinates of the two diagrams together we derive the Combined Shearing Force Diagram H a b c d e 5 6 f, &c., of Fig. 19.

Draw the Bending Moment Diagram (L n M, Fig. 19) for the concentrated loads as described for Fig. 17. Then draw on the opposite side of L M a parabola, having its axis bisecting L M at right angles, and the ordinate at the centre of L M equal to the Maximum Bending Moment due to the uniform load. This

ordinate must be measured to the same scale as that of the ordinates of the concentrated Bending Moment Curve. The combined ordinate measures the Combined Bending Moment.

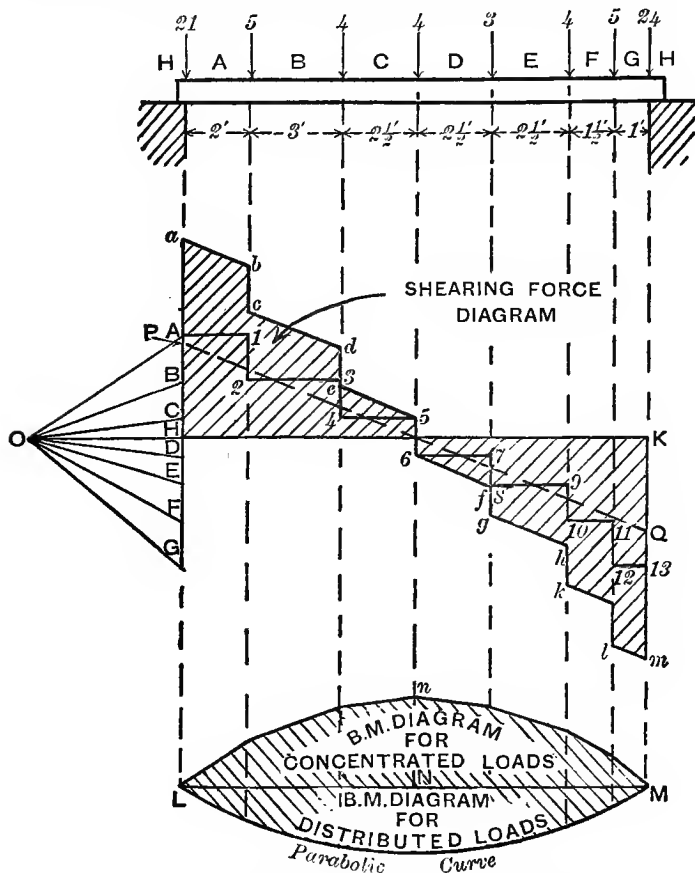
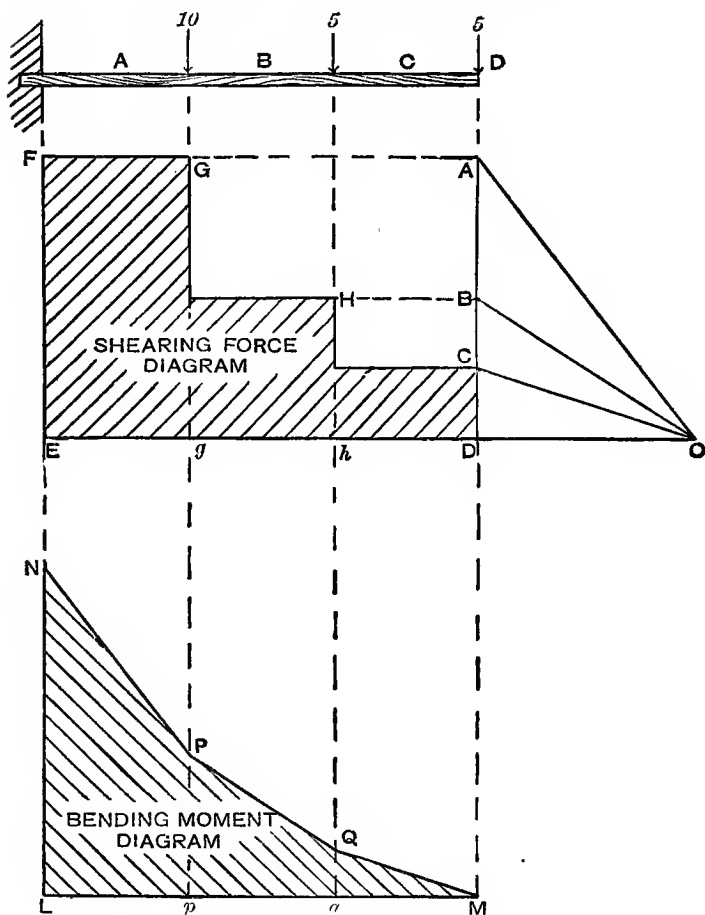


FIG. 19.—BEAM WITH UNIFORM AND CONCENTRATED LOADS.

EXAMPLE II.—A cantilever 15 feet long has a load of 5 tons at its outer end, 5 tons at 5 feet from it, and 10 tons at a point 10 feet from the end. Find graphically the diagrams of shearing force and bending moment.

ANSWER.—The upper part of the figure shows the cantilever

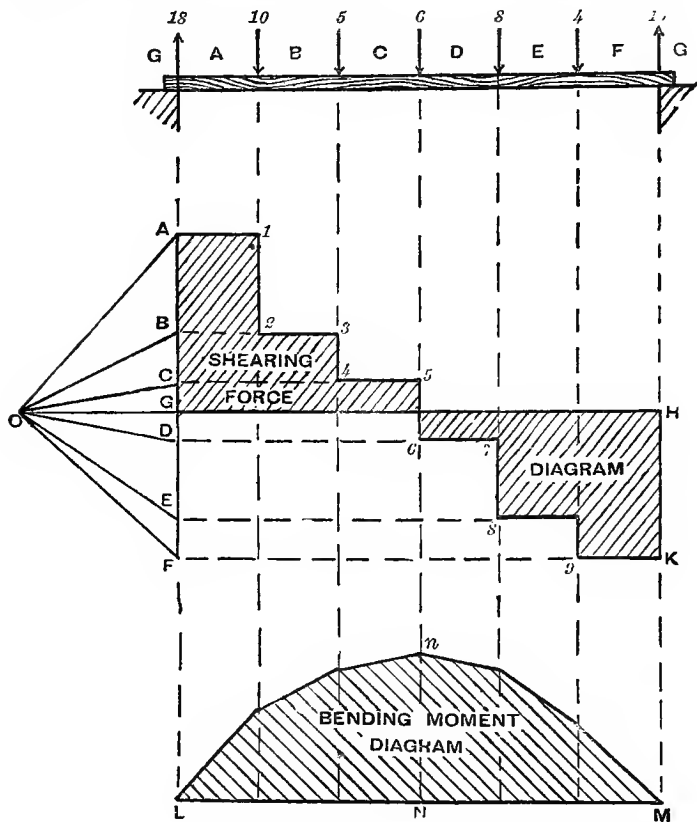
and the positions of the loads. Project down from these positions and the inner end of the beam, and then set out $AB = 10$ units, $BC = 5$, and $CD = 5$, to represent the forces A , B , C ,



CANTILEVER IN EXAMPLE II.

and OD respectively. Draw horizontal lines through A , B , C , and D to intersect the lines of the forces. This gives us the Shearing Force Diagram as shown shaded.

To obtain the Bending Moment Diagram take any point O in ED , and join it to A , B , and C . Then take a base line ML parallel to OD , and draw MQ in the space C parallel to OC , QP in the space B parallel to OB , and PN in the space A parallel to OA . Then $LMQPN$ is the Bending Moment



BEAM IN EXAMPLE III.

Diagram, the scale being that adopted for the shear multiplied by the length of OD measured on the scale employed in setting out the length of the beam.

EXAMPLE III.—A beam of 12 feet span carries five loads equally spaced along its length, the first and last being each 2

feet from the nearest end. The values of the loads are 10, 5, 6, 8, and 4 tons respectively. Obtain graphically diagrams showing the shear and bending moment at every point of the beam.

ANSWER.—In this case we shall determine the reactions by calculation, thus:—

Reaction at left hand due to A B is $\frac{5}{8} \times 10 = 8\frac{1}{8}$ tons.

“ “ “ B C “ $\frac{4}{8} \times 5 = 3\frac{1}{8}$ “

“ “ “ C D “ $\frac{3}{8} \times 6 = 3$ “

“ “ “ D E “ $\frac{2}{8} \times 8 = 2\frac{1}{2}$ “

“ “ “ E F “ $\frac{1}{8} \times 4 = 0\frac{3}{8}$ “

∴ Total left-hand reaction G A = 18 “

The whole load is 33 tons, and therefore the right-hand reaction must be 33 – 18, or 15 tons.

We can now proceed as before, making G A = 18, A B = 10, B C = 5, &c., and drawing horizontal lines through A, B, C, &c., to obtain the Shearing Force Diagram.

Take a point O in the horizontal through G, and join it to A, B, C, &c.

Then the part of the Bending Moment Diagram in the space A is parallel to O A, in the space B to O B, in C to O C, and so on, as in Example II.

We might, of course, have determined the reactions from the Funicular Polygon L n M N in the first instance; but had we done so we would probably not have got the line O G horizontal, and would have had to redraw it as explained in the text.

Centre of Gravity of an Area.—Divide the area into elements, such as parallelograms, triangles, &c., the centres of gravity of which can be easily determined.

If the area is bounded by a curved line, divide it into very narrow strips, so that they may be considered approximately as parallelograms.

We have divided the area shown in Fig. 20 into three rectangles, and have found the centre of gravity of each. We first, assume a line lying in any direction, such as the line X X, along which the pull of gravity acts. The centre of gravity of each area is a point in the line of action of the pull of gravity on that area. The line of action of gravity will be parallel to this assumed line X X. The way may be towards either the left or the right as may be found most suitable, and the magnitude will be proportional to the area. The forces B C, C D, and D E represent completely the action of gravity on the top, the centre, and the bottom rectangles respectively.

Proceed to find the resultant of the three forces B C, C D,

and D E, as explained for Fig 1a. This is shown in Fig. 19 by the Polar Diagram B C D E O, and the corresponding funicular polygon 1 2 3 4. B C, C D, and D E, in the Polar Diagram are proportional to the areas of the three rectangles.

The line of action of the resultant of the three forces B C, C D, and D E passes through the centre of gravity of the whole area. This line is represented by the line 4—M. Now, assume a line at right angles to X X as a line along which the pull of gravity acts. Proceed in exactly the same way with regard to this line as has been done for the line X X, and we obtain another line passing through the centre of gravity of the

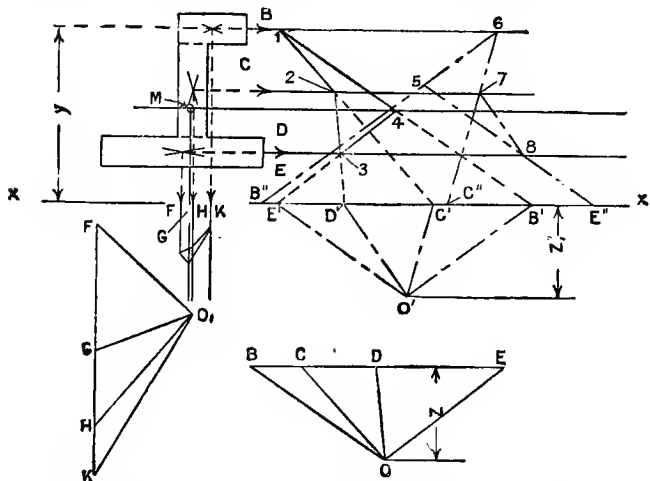


FIG. 20.—CENTRE OF GRAVITY AND MOMENT OF INERTIA.

whole area. The intersection M of these two resultants gives the centre of gravity of the whole area. The forces in the second case are called F G, G H, and H K, and the Polar Diagram F G H K O₁, with its corresponding Funicular Polygon, is shown in the figure.

Moment of Inertia of an Area.—If we wish to find approximately the Moment of Inertia round the line X X of the area in Fig. 20, we must first of all divide the area into elements just as in finding the centre of gravity. Then proceed to draw the polar diagram B C D E O and the corresponding funicular polygon 1 2 3 4.

Now consider the top polygon and how we may determine its Moment of Inertia.

Produce the two lines derived from the polar diagram, which meet in the line of action of the pull of gravity on that area (viz., 1—2 and 1—4) until they intersect the line XX in the points B' , C' . Do the same for the lines 1—2 and 2—3, and 2—3 and 4—3, which meet on the lines of action of the pull of gravity on the middle and bottom areas. These lines intersect XX in the points C' , D' , and D' , E' , respectively.

Consider $B'C'$, $C'D'$, and $D'E'$ as the magnitudes of the forces acting along the lines BC , CD , and DE respectively. Proceed as if to find their resultant by drawing the polar diagram $B'C'D'E'O'$ and the corresponding funicular polygon 5 6 7 8.

Produce, as before, the lines which meet in BC (viz., 6—5 and 6—7) to intersect the lines XX in the points $B''C''$. Do the same for the lines 5—8 and 7—8, or, as we have done in the figure, produce the one which will cut XX in a point furthest from B'' . $B''C''$ measures to a certain scale the moment of inertia of the top area round the line XX , and $B''E''$ the moment of inertia of the whole area round the same line. Greater accuracy would be obtained by dividing the area into smaller elements.

Proof.—Let y in Fig. 20 represent the distance the centre of gravity of the top area is from the line XX .

Now, since the two triangles BCO and $B'O'1$ are similar:—

$$BC : Z :: B'C' : y.$$

Then,
$$B'C' = \frac{BC \times y}{Z}.$$

Again, the two triangles $B'C'O'$ and $B''C''6$ are similar:—

Hence,
$$B'C' : Z_1 :: B''C'' : y.$$

And,
$$B''C'' = \frac{B'C' \times y}{Z_1}.$$

Substituting the above value of $B'C'$ we get:—

$$B''C'' = \frac{BC \times y^2}{Z \times Z_1}.$$

But, $BC \times y^2$ is the moment of inertia for the top area with respect to the line XX , provided the depth of the area is small in comparison with y . $B''C''$ measured with the scale called the "area scale," as used for drawing BC (in order to represent the area of the top rectangle), gives the value of this moment of

inertia, if Z and Z_1 are $\frac{1}{2}$ unit of the scale which is used for setting off the lengths in drawing the section.

SCALE FOR MEASURING THE MOMENT OF INERTIA.—Subdivide the unit of the scale used for representing the areas, into as many divisions as is represented by the number found by multiplying Z and Z_1 , which are both measured by the length scale. One of these subdivisions will be the unit for the Moment of Inertia Scale. Or, measure $B''E''$ with the area scale and multiply the reading first by Z and then by Z_1 .

Engine Mechanism.—In the Frame Diagram, Fig. 21a, the bars BC and OE represent the centre lines of the piston-rods of a compound engine the heads of which are guided in parallel straight lines. The bars AC and DF are short

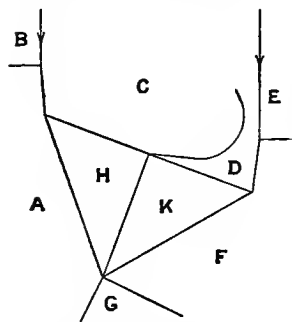


FIG. 21a.—FRAME DIAGRAM.
ENGINE MECHANISM.

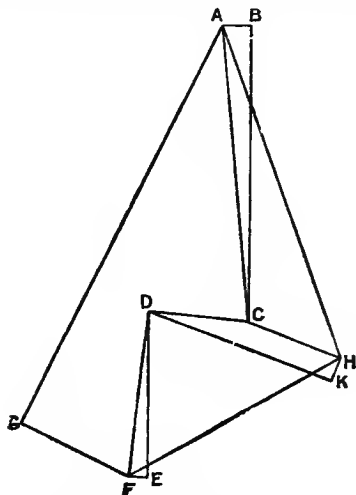
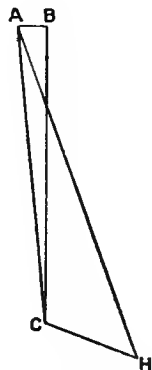


FIG. 21b.—STRESS DIAGRAM.



STRESS DIAGRAM FOR
JOINT ACH.

connecting-rods, driving the crank FG , by means of the triangular frame shown. The joint $CDKH$ is constrained to

move in an arc of a circle round a point in the bar C D produced towards the right. The bar C D is called a radius rod. The lines of action of all the external forces acting in the structure are shown. A B and E F are the guide pressures, C D the push or pull in the radius rod, F G the push or pull in the crank, and G A the crank effort or tangential resistance.

First Method.—We commence the Stress Diagram, Fig. 21*b*, by drawing D E to represent in magnitude the total pressure on the right-hand piston-rod. Then, E F and D F fix the point F, while F K and D K determine K.* But, we can get no further until we draw the Stress Diagram for the joint A C H. This is done by drawing B C to represent to the same scale as before the total pressure on the left-hand piston-rod. Then, the points A and H are determined.

We must now fit the Stress Diagram for the joint A C H, to the Stress Diagram already drawn; so that the point C shall lie on the line drawn through D parallel to the bar C D and the point H on the line drawn through K parallel to the bar K H, C H being kept parallel to the bar C H. Then the Stress Diagram, Fig. 21*b*, can be completed in the usual way.

Second Method.—Find the forces acting in A C and D F, and then find their resultant. Produce the line of action of this resultant to cut the line of action of the force C D; when, by joining this point with the crank pin, we get the line of action of the resultant force acting on the said crank pin. Finally, draw the Stress Diagram from the supplementary data.

The following is a list of books and papers on Graphic Statics and the Design of Structures:—

The Design of Structures, Bridges, Roofs, &c., by S. Anglin, C.E. (Chas. Griffin & Co., London, 1895.)

A Practical Treatise on Bridge Construction, by Prof. T. Claxton Fidler. (Chas. Griffin & Co., London.)

Graphical Determination of Forces in Engineering Structures, by James B. Chalmers, C.E. (Macmillan & Co., London.)

Graphic and Analytic Statics, by Robert Hudson Graham, C.E. (Crosby Lockwood & Co., London.)

Graphics, by Prof. R. H. Smith, M.Inst.M.E. (Longmans, Green & Co., London.)

Mechanics, vol. ii., by A. Jay Du Bois, C.E., Ph.D. (Chapman & Hall, London.)

Applied Mechanics, by Gaetano Lanza. (Chapman & Hall, London.)

Theory of Structures and Strength of Materials, by Henry T. Bovey, M.A., D.C.L. (Chapman & Hall, London.)

Graphic Methods of Computing Stresses in Jointed Structures. Paper by C. O. Burge, Proc. Inst. C.E. Vol. lxxiv., p. 192.

Graphic Methods of Engine Design, by A. H. Barker. (The Technical Publishing Co., Ltd., Manchester.)

* The line F K has been omitted in the diagram.

Mechanical Graphics, by G. Halliday. (London, 1889.)

Elements of Graphic Statics, by K. von Ott, translated by G. S. Clark. (E. & F. N. Spon, London, 1888.)

Principles of Graphic Statics, by G. S. Clark. (E. & F. N. Spon, London, 1888.)

Elements of Graphic Statics, by L. M. Hoskins. (Macmillan & Co., London, 1892.)

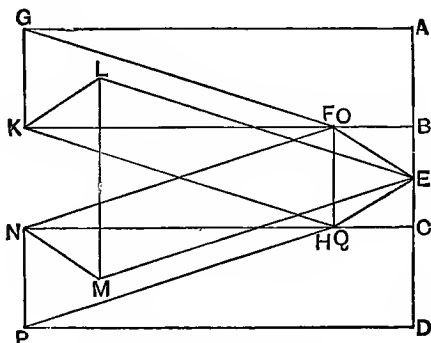
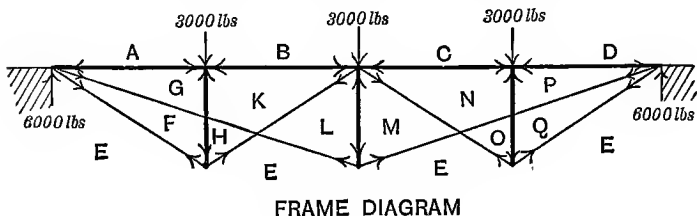
Economics of Construction, by Robt. H. Bow. (E. & F. N. Spon, London.)

Applied Mechanics, 2nd edition, by Prof. James H. Cotterill. (Macmillan & Co., London.)

Re Funicular Polygon at pp. 84 and 108.—The word funicular, as used in mathematics, is an epithet for a curve, which is the same as the catenary; and also for a polygon hanging freely by its extremities. The Funicular Polygon or Link Polygon is really a Stress Diagram drawn from a Frame, such as that shown by Fig. 13 on p. 11. When forces in equilibrium act at the corners of a series of links or bars jointed together at their extremities, the force acting along each link can be readily found by a special application of the triangle of forces, as in the above-mentioned case.

LECTURE V. — QUESTIONS.

1. Draw the following Frame Diagram to twice the given scale. Verify the accompanying Stress Diagram by the first and by the second methods given in this lecture, with accurate drawings. Use red or distinctive dotted lines for the second method, if you give only one figure for the two stress diagrams. Give a neat table of the members in compression and in tension, and correct the stated values of the reactions at each end of the truss.



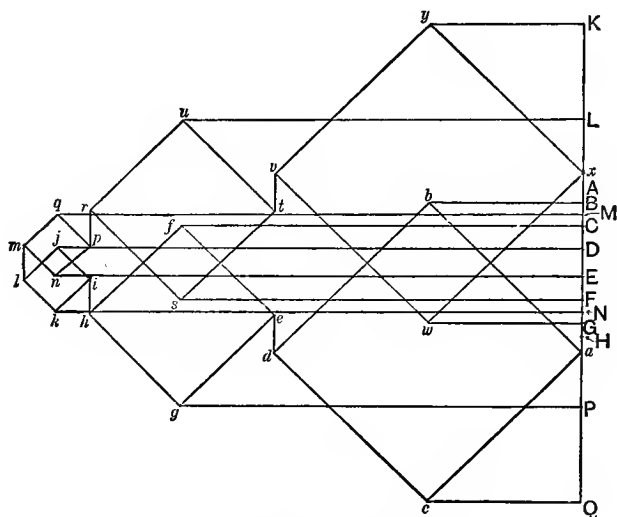
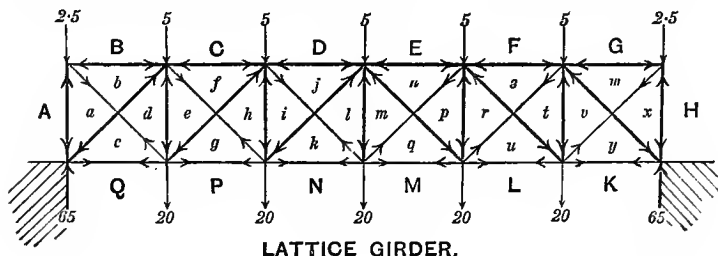
FINK TRUSS FOR QUESTION 1.

2. A triangular frame is at rest under the action of three external forces. Prove that a certain diagram will represent the stresses in the bars of the frame. Extend this proposition to the case of a lattice girder of the Warren construction with four bays in the lower boom and three bays in the upper boom, loaded at the centre of the lower boom and supported at the ends, giving the Stress Diagram and showing how to distinguish the portions which are in compression or extension.

3. A triangular frame is acted on by three forces applied at its respective angular points and in equilibrium; investigate a method of constructing the diagram of all forces brought into play. Taking the case of a frame on the principle of the Warren Girder having four bays in the lower boom and three in the upper boom, and loaded at the centre of the lower member with a weight W , explain the method of constructing the diagram of forces,

drawing the same, and distinguishing those bars which act as struts from those which act as ties.

4. The lower boom of a Warren Girder, supported at both ends, is divided into three bays. The upper boom has two bays, and the bracing bars are each inclined at 60° to the horizon. Find by graphic construction the stresses in the several pieces when the frame is loaded with 1,000 lbs. at the middle of the top boom.

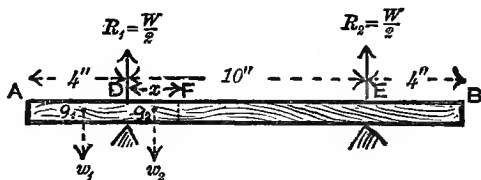
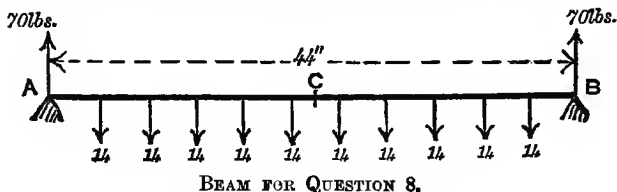


5. A Warren Girder has five bays consisting of equilateral triangles. If it be supported at each end and loaded at the two bottom central joints with loads of 18,000 lbs., find graphically the stress on each member, and show whether it is tensile or compressive. Explain fully the reasons and theory of the method you employ in obtaining your result.

6. A Warren Girder of six bays with equilateral bracing, each bay being 10 feet long, is loaded with a distributed weight of 1 ton per foot run placed along the top of the girder; obtain the diagram of stress, and calculate the stresses in the various members, assuming that $\cotan. 60^\circ = .577$, cosec. $60^\circ = 1.155$. *Ans.*

7. A lattice girder is loaded in the manner shown by the foregoing figure. Draw the Stress Diagram by the method explained in the text, and see if you get the same results as shown.

8. A bar of pine 44 inches long rests on props at its extremities, and just supports 10 weights, of 14 lbs. each, hung at equal intervals of 4 inches along the rod. Find graphically the B M at the centre of the bar and the amount of a single weight, which, if hung at the centre of the bar, would stress it to the same extent (see figure). *Ans.* 76.3 lbs.



9. A horizontal uniform bar, 18 inches long, is laid over two supports, each 4 inches from its ends, as shown in the figure. Find graphically two points at which the bending moment is zero, the bar being loaded by its own weight (see figure). *Ans.* 2 inches from inside of supports.

10. Given an iron arched rib, hinged at both ends, and a system of vertical loads, show how we find the stress at any point of any section. Prove the rule for stress at any point of a section when we know the resultant of all the forces acting on the structure on one side of the section.

11. A beam, ABCDE, has a vertical supporting force at A; at E there is a pin joint support. AB is 5 feet, BC is 2 feet, CD is 6 feet, DE is 4 feet. There are vertical loads of 2 tons at B and 3 tons at D, and at C there is a load of 5 tons inclined at 30° to the vertical, its horizontal component being towards A. All forces in one plane. Find the supporting forces, graphically or otherwise. (S. & A. Adv. Exam., 1897.)

12. Suppose the vertical loads and supporting forces of a horizontal beam to be known, show how we find (1) the total shearing force at any section, (2) the position of the neutral line, (3) the tensile or compressive stress at any place. Prove your statements. (S. and A. Adv., 1899.)

13. In calculating by graphical methods the tensile or compressive forces in the several members of braced girders, why is it usual to

represent the weight of, or upon, the several bars, by equivalent forces acting at the joints, and to assume that the joints are frictionless pin joints or hinges? A Warren girder has 6 equal bays on the bottom flange, the girder is 90 feet span and 13 feet deep. It is loaded with a weight of 20 tons resting on the apex of the top flange, which is 37 feet 6 inches from the left abutment. Determine either graphically or analytically the forces in the diagonals and flanges of the girder produced by this load.

(S. and A. Adv., 1899.)

14. A railway bridge over a road is 40 feet span. An engine with its tender stands upon the bridge. The weights on the leading, the driving, and the trailing axles of the engine are 9, 15, and 7 tons respectively, while the load on the three axles of the tender is 7 tons on each. The engine stands so that the leading axle is $2\frac{1}{2}$ feet from the end of the bridge, and the distance between the centres of the engine axles is 8 feet, and between the tender axles is 4 feet 6 inches, while between the trailing axle of the engine and the leading axle of the tender is 8 feet. Draw the bending moment and shearing force diagrams for the above position of the engine, and write down the maximum value of the bending moment and also the value and position of the maximum shearing force.

(S. and A., Adv., 1899.)

N.B.—See Appendices B and C for other questions and answers.

LECTURE V.—A.M.INST.C.E. EXAM. QUESTIONS.

1. A framed girder, 8 feet deep, supported at the ends, with double triangulation, has the bracing bars inclined at 45° . The girder has seven bays, each 8 feet long, and carries at each joint of the bottom boom a load of 3 tons and at each top joint a load of 6 tons. Find the total stress in each of the inclined bracing bars and in the top boom in the second bay from the left end. (I.C.E., Oct., 1897.)

2. A plate-girder is required to carry a fixed load of 1 ton per foot run and a rolling load of $1\frac{1}{2}$ ton per foot run. Clear span 75 feet. Draw the diagrams of maximum bending moment and maximum shearing force. Curves may be sketched in if three or four points are determined. (I.C.E., Oct., 1897.)

3. Design a centre section for the girder in the previous question, taking the depth of girder at 8 feet, and the working stresses at 4 tons per square inch of gross section in compression, and 5 tons per square inch of net section in tension. Assume thickness of plates and size of rivets as you think best. (I.C.E., Oct., 1897.)

4. The platform of a bridge 160 feet span is carried by cross-girders resting on the lower joints of a pair of Warren girders (the members inclined at 60°). Assuming eight bays, find the stress on each member of the girders due to a uniform load of 4 tons per foot run covering the platform. (I.C.E., Oct., 1899.)

5. If a uniformly distributed travelling load of 4 tons per foot run traverse the bridge of the preceding question, describe how the stress due to it on the centre diagonals of the Warren girders varies during the passage, and find the magnitude of that stress when the load extends from one end to the centre. Compare it with that due to the fixed load. (I.C.E., Oct., 1899.)

6. A cantilever girder bridge consists of a central girder span $2a$, the extremities of which are jointed to the ends of cantilevers of length c projecting from the piers. The bridge is uniformly loaded throughout the span; find the straining actions at any point, and explain how they are reduced by the cantilever construction as compared with a simple girder bridge of the same total span. (I.C.E., Oct., 1899.)

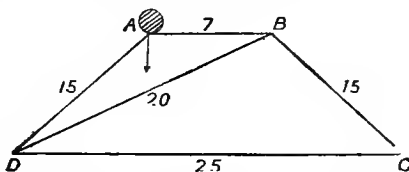
7. A beam is strengthened by the addition of tension rods below attached to the lower end of a vertical strut supporting the centre, the whole forming a simple triangular truss of given span and depth. Find the stress on each member of the truss when loaded with a weight in the centre. (I.C.E., Oct., 1899.)

8. Find the centre of gravity of the area in one corner of an equilateral triangle cut off by its inscribed circle. (I.C.E., Feb., 1900.)

9. A beam is strengthened by the addition of tension rods below attached to the lower ends of two vertical struts dividing the span into three equal parts, the whole forming a simple trapezoidal truss. Find the stress on each member when the beam carries a uniformly distributed load, and describe the effect of a load which is not uniformly distributed. (I.C.E., Feb., 1900.)

10. Explain how you would find experimentally, and also by calculations the centre of gravity of a uniform board of any given shape. A ship, with its equipment, weighs 6,000 tons. How far will its centre of gravity move if a gun weighing 30 tons is moved a distance 20 feet across the deck. (I.C.E., Feb., 1901.)

11. A light framework, consisting of four jointed rods in the form of a trapezoid $ABCD$, and a fifth rod connecting D and B , and having AB parallel to CD , is supported at D and C with DC horizontal. It is now loaded with 100 lbs. at A ; find the pressures on the supports at D and C and the stresses in the various rods. The dimensions are $AB = 7$, $BC = AD = 15$, $DB = 20$, $DC = 25$.



(I.C.E., Feb., 1901.)

12. A beam, $ABCDE$, has a vertical supporting force at A . At E there is a pin-joint support. AB is 5', BC is 2', CD is 6', DE is 4'. There are vertical loads of two tons at B , and 3 tons at D , and at C there is a load of 5 tons inclined at 30° to the vertical, its horizontal component being towards A . The beam is symmetrical in section and weighs $\frac{1}{2}$ ton. All the forces are in one plane. Find graphically the supporting force at A and the reaction of the pin-joint support at E . (I.C.E., Oct., 1901.)

13. The cross section of a cast-iron girder has the following dimensions: total depth 12 inches, top flange 3 inches by 1 inch, bottom flange 9 inches by 2 inches, thickness of web $1\frac{1}{2}$ inch. Assuming the web to be of uniform thickness throughout, and all the corners and edges to be square, find graphically the position of the "centre of area," or "centre of gravity" of the section. (Draw the section half-full size.) (I.C.E., Feb., 1902.)

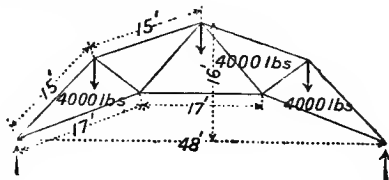
14. A continuous girder consists of two spans. One span of 100 feet is loaded with $1\frac{3}{4}$ tons per foot run, the second span of 80 feet is loaded with $2\frac{1}{2}$ tons per foot run. Find the values of the supporting forces, and the maximum bending moment for the whole girder. Both ends of the girder are free. (I.C.E., Feb., 1902.)

15. Find, by graphic construction, the centre of gravity of a section of an I beam, top flange 4 inches by 1 inch, web, between flanges, 14 inches by $1\frac{1}{2}$ inch, bottom flange 9 inches by 2 inches. (I.C.E., Oct., 1902.)

16. A parabolic girder has 4 bays, verticals from the intersection of the straight members of the upper chord support the cross-girders: the upper ends of the two verticals at quarter and three-quarters the length are connected by braces to the feet of the centre vertical, show that the length of these braces is a measure of the maximum stress they will have to sustain, on the same scale that the length of the lower chord represents the horizontal stress when the live load covers the span. (I.C.E., Oct., 1902.)



17. A roof, as in sketch, is loaded as shown; find the stresses in its members. (I.C.E., Oct., 1902.)



18. A girder 100 feet long is supported 20 feet from each end, and carries a uniform load of 2 tons per foot run; draw the diagrams of bending moment and shearing force. (I.C.E., *Oct.*, 1902.)

19. A Warren girder, length 100 feet, is divided into five bays on the lower flange, the length of the inclined braces being 20 feet; if loads of 30 tons are carried by the girder at the joints 20 feet and 40 feet from one end, find the stresses in the members. (I.C.E., *Feb.*, 1903.)

20. If a uniform live load moves over the bridge in the previous question, in what positions of it would maximum stresses in the various members be produced? (I.C.E., *Feb.*, 1903.)

21. A girder 100 feet long is supported at each end and in the middle, and carries a uniform load of 2 tons per foot run. Draw the bending moment and shearing force diagrams, and find the pressure on each support. (I.C.E., *Feb.*, 1903.)

N.B.—See *Appendices B and C* for other questions and answers.



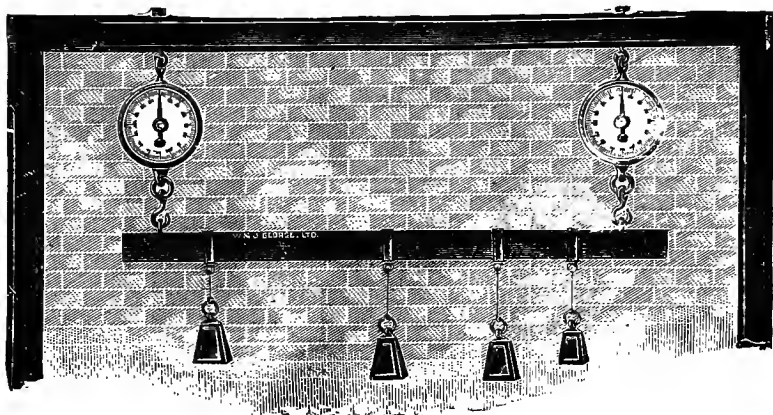
LECTURE VI.

CONTENTS.—Parallel Forces—Pressures and Reactions from the Supports of Loaded Beams—Example I.—Bending Stresses, Neutral Surface and Neutral Axis—Bending Moments and Shearing Forces on Cantilevers and Beams—Cantilever loaded with a single concentrated weight at its outer end—Cantilever loaded with three concentrated loads—Cantilever with load distributed uniformly over its whole length—Cantilever with a uniformly distributed load over its whole length together with a concentrated load acting at the free end—Cantilever with a uniformly distributed load over part of its length—Beam supported at both ends and loaded with a single concentrated load at the centre of the span—Beam supported at both ends and loaded with a single concentrated load in any position—Beam supported at both ends and loaded with several separate concentrated loads—Beam supported at both ends and loaded throughout its length with a uniformly distributed load—Example II.—Beam carrying a uniformly distributed load over the whole of its length and a single concentrated load—Beam supported at both ends and loaded uniformly over a portion of its length not extending to either abutment—Graphic Construction of the Parabola upon a given base, while the height is obtained by calculation, in the case of a beam carrying a uniformly distributed load—Questions.

Parallel Forces.—The parallelogram of forces enables us to find the resultant of two forces which intersect; but, since parallel forces do not intersect, we are unable to apply that construction to determine the resultant of any two parallel forces. We can, however, find this resultant very simply by other means. Hence, to explain the method of doing so, let us take a beam supported at its extremities and loaded in the middle. Then, not only the weight of the beam, but also the load produce pressures on the supports, and it is evident that they will bear it equally. If the weight be placed to either side of the centre, we can easily understand that the support nearer to the weight must bear more than the remote one. But, it is not so easy to see in what proportion, the weight is divided between the supports. Students may refer to Lecture III., Vol. I., before studying the following propositions.

Pressures on and Reactions from the Supports of Loaded Beams.—Let us employ the apparatus shown in the accompanying figure. An iron or graduated wooden bar is supported by means of eye-bolts from the hooks of two spring balances, as shown in the figure. These hooks are exactly one metre apart. Before any of the four weights is suspended from the beam, the

spring balances show a pressure of so many units to the weight of the beam itself. If we divide the length of the bar into as many equal parts as there are pounds in the single weight which we shall now suspend from it, then the additional pressure at *one* end as registered by the nearer spring balance will be represented by the number of divisions that the weight is distant from the other end. For example, suppose we place a weight of 4 lbs. at $\frac{1}{4}$ of the distance between the balances from the *left*-



EXPERIMENTAL APPARATUS FOR DETERMINING THE REACTION AT THE SUPPORTS OF A HORIZONTAL BEAM, WHEN EQUALLY AND UNEQUALLY LOADED.

hand support, then we shall find from the nearer spring balance that the *additional pressure* is 3 lbs. But, the weight is then exactly $\frac{3}{4}$ of the whole distance from the other end, consequently the other balance will indicate 1 lb. of additional pull.

Let us suspend, two, three or four weights from the beam as shown in the figure. We must now find the pressures which each weight taken separately produces according to the above rule. These results are then added together, and to half the weight of the beam, to give the total pressures on the two balances. The principle involved in these calculations is of the very greatest importance.

EXAMPLE I.—A uniform beam, 12 feet long and weighing 100 lbs., is supported at both ends, and carries a weight of 2 cwt. at a distance of 3 feet from one end; find the pressure on each point of support.

Take moments round B, then

$$R_1 \times AB = W \times DB + wL \times CB$$

$$R_1 \times 12' = 224 \times 9' + 100 \times 6$$

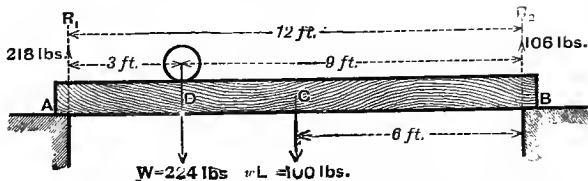
$$\therefore R_1 = \frac{2616}{12} = 218 \text{ lbs.}$$

To find R_2 , we get

$$R_1 + R_2 = W + wL$$

$$\therefore R_2 = 224 + 100 - 218$$

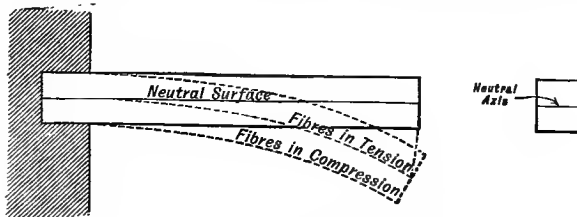
$$\text{Or, } R_2 = 106 \text{ lbs.}$$



UNIFORM BEAM SUPPORTED AND LOADED AS SHOWN.

Bending Stresses, Neutral Surface and Neutral Axis.—

If a cantilever be loaded, either with a uniformly distributed load over its whole length or a single concentrated load acting at its free end, then the fibres or resisting material of which the beam is composed will be subjected to a *tensile stress* tending to stretch or elongate the *upper layers*; whilst those in the *lower half* will be subjected to a *compressive stress* tending to compress or crush the fibres. These actions are graphically represented by

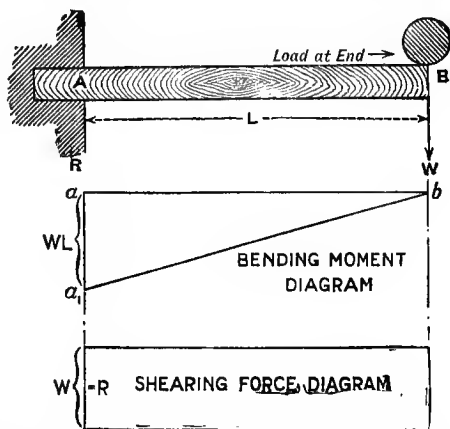


NEUTRAL SURFACE AND NEUTRAL AXIS IN A CANTILEVER LOADED AT OUTER END.

the accompanying figure. There must therefore be one layer or horizontal longitudinal section which is neither in tension nor in compression. This surface is known as the *neutral surface*, and its intersection on any transverse section, is called the *neutral axis* of that section, as shown by the small figure on the right hand.

If a beam be supported at both ends and loaded anywhere between the bearings (as shown by the first and second figures in this Lecture), then the upper set of layers are naturally in compression and are shortened, whilst the under set are in tension* and are lengthened so that the centre of its curving is above the beam. In the above cantilever the reverse takes place.

Bending Moments and Shearing Forces on Cantilevers and Beams.—From the previous article we shall consider the bending moments in the cantilever as *negative*, and the bending moments in the loaded beam, which is supported at both ends, as



(1) CANTILEVER LOADED WITH A SINGLE CONCENTRATED WEIGHT AT ITS OUTER END.

positive. We shall see from the illustrations which follow, that in continuous beams the bending moments are partly positive and partly negative. Also, a shearing force at any particular section will be considered *positive* when it tends to shear the *right-hand* portion of the beam in an upward direction.

The bending moment at a given section of a loaded cantilever is equal to the sum of the moments about that section of all the loads between it and the free end of the cantilever; whilst, the shearing force at any section is equal to the sum of the loads between the section and the free end of the cantilever.

*It is therefore clear that a wooden beam may have a saw-cut inserted into the upper set of layers without very materially affecting the strength of the beam, if the edges come together and jam up the saw-cut in the bending of the beam; or, if a wedge be inserted into the cut so as to transmit the compressive stresses. A saw-cut, however, in the lower side would greatly affect the strength of the beam.

The bending moment at a given section of a beam, which is acted on by external loads, is equal to the algebraic sum of the moments of all the external forces acting on the portion of the beam on either side of it; and the shearing force at any section is the algebraic sum of all the external forces acting on either portion of the beam into which the section divides it.

In all cases where the loading is not continuous, but changes abruptly at one or more points of the span, it is necessary to consider separately each portion of the beam between the points of discontinuity.

In representing bending moments and shearing forces graphically by the aid of diagrams; they are each considered *positive* when measured above a horizontal datum line; and *negative* when measured downwards from this line.

It is also well to note, that the ordinates in the bending-moment diagram represent either lb.-inches, lb.-feet or ton-feet, whereas the ordinates in the shearing-force diagram represent lbs., cwts. or tons.

(1) **Cantilever Loaded with a Single Concentrated Weight at its outer end.**—Let the load W be placed at the outer end B of a cantilever of length L which is fixed at the end A into the wall at right angles, as shown by the previous figure.

The Maximum Bending Moment at A is $B.M._A = -W \times L$,

and The Maximum Shearing Force at A is $S.F._A = -W = -R$.

This maximum bending moment occurs at the point of support. If the cantilever was of uniform section and material; then, it would be broken close to the wall by a load sufficient to overcome the moment of resistance of the cantilever.

The diagram of bending moment is a triangle, where $aa_1 = WL$, and the moment at any section is represented by the vertical ordinate at that point,

The diagram of shearing force is a rectangle, since *all* ordinates are equal to W , or also equal to R in this case.

(2) **Cantilever loaded with three concentrated loads.**—Let AB be the cantilever fixed at end A. It is loaded at points C, D, B with weights W_1, W_2, W_3 , which are acting at distances x_1, x_2, x_3 respectively from the fixed end as in the next figure.

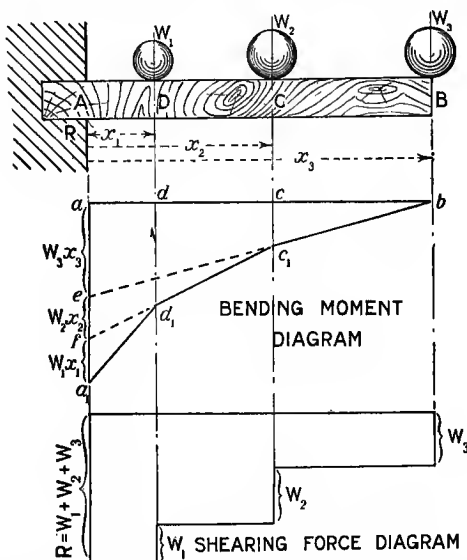
The maximum bending moment occurs at the fixed end A of the cantilever, and is equal to

The Maximum $B.M._A = -(W_1x_1 + W_2x_2 + W_3x_3)$.

The maximum shearing force at the fixed end A is equal to

The Maximum $S.F._A = -(W_1 + W_2 + W_3) = -R$.

To draw the diagram of bending moments.—Let the horizontal line $a b$ represent the cantilever. Draw $a e$ at right angles to $a b$ and let its length represent, on a scale of moments, the moment of W_3 about the fixed end A, i.e., $W_3 x_3$. Produce $a e$ to f , making ef represent the bending moment $W_2 x_2$; and, similarly fa_1 represent the bending moment $W_1 x_1$ on the same scale of bending



(2) CANTILEVER LOADED WITH THREE CONCENTRATED LOADS.

moments. Now, join e to b , and let c_1 be the point on $b e$, vertically below W_2 . Join f to c_1 and let d_1 be the point on fc_1 vertically below W_1 ; then join a_1 to d_1 . The polygon $a b c_1 d_1 a_1$ will represent the diagram of moments for the whole cantilever. The bending moments at any section of the loaded cantilever will be found, by measuring on the scale of bending moments the length of the ordinate of the polygon drawn vertically below the point which is being considered.

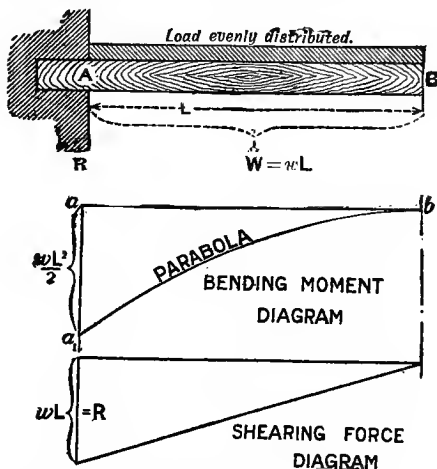
The shearing force diagram for this loaded cantilever is shown underneath the bending moment diagram. In order to find the shearing force at any section of the loaded cantilever, you draw

NOTE.—When a cantilever is loaded with one or more concentrated weights, the shearing force or stress at any section is equal to the sum of the weights between the section and the free end of the cantilever.

a vertical line downwards from the section, and measure upon the shearing force scale, the length of the vertical ordinate of the stepped diagram at that section.

(3) **Cantilever with load distributed uniformly over its whole length.**—Let L be the length of the cantilever, and w the uniformly distributed load per unit of length, *i.e.*, per foot run. In this case, the total load $W = wL$, and this weight W may be assumed as acting at its centre of gravity.

It will be seen, that the bending moment at any section



(3) CANTILEVER WITH LOAD DISTRIBUTED UNIFORMLY OVER ITS WHOLE LENGTH.

which is distant x feet from the free end of the uniformly loaded cantilever is,—

$$\text{B.M.}_x = -wx \times \frac{x}{2} = -\frac{wx^2}{2}.$$

Hence, taking moments about the edge of the support where $x = L$, we get

$$\text{The Maximum B.M.}_A = -wL \times \frac{L}{2} = -\frac{wL^2}{2}.$$

Since the bending moment varies as the square of the distance of the section under consideration from the free end of the cantilever, the successive bending moments may easily be represented by the ordinates of a parabola, of which the free end of the can-

tilever may be taken as the vertex (see my *Elementary Manual of Applied Mechanics*, Lect. XXI., for the properties of a parabola).

The shearing force at any section which is distant x feet from the free end of the uniformly loaded cantilever is,—

$$\text{S.F.}_x = -wx.$$

The maximum value of the shearing force occurs at the fixed end, and its value is,—

$$\text{The Maximum S.F.}_A = -wL = -W = -R.$$

To draw the diagram of bending moments.—Let ab represent the cantilever.

At a draw a vertical line downwards and make the length aa_1 on any scale of bending moments to represent $\frac{wL^2}{2} = \frac{WL}{2}$. Now draw a parabola to pass through the point a_1 and to touch ab at b . The bending moment at any section of the uniformly loaded cantilever is found by measuring the length of the ordinate to the parabola ba_1 , to scale at that point.

The diagram of shearing force for the uniformly loaded cantilever is a triangle as shown by the figure.

This cantilever would also break close to the wall if made of uniform section and material. But, as we see, it would sustain double the load of the first case, where the concentrated load was acting at the free end of the cantilever. Therefore, since *the bending moment is only half* what it is in the first case, it will take twice the load in this instance to break the cantilever close to the wall.

(4) Cantilever with a uniformly distributed load over its whole length together with a concentrated load acting at the free end.

Let W be the weight acting at the free end, and w be the uniform load per unit of length of the cantilever.

Then, the bending moment at a section distant x feet from the free end is,—

$$\text{B.M.}_x = -\left(Wx + \frac{wx^2}{2}\right).$$

The maximum bending moment occurs at the fixed end of the cantilever, where $x = L$.

$$\text{The Maximum B.M.}_A = -\left(WL + \frac{wL^2}{2}\right).$$

Also, the shearing force at any section of the loaded cantilever is,—

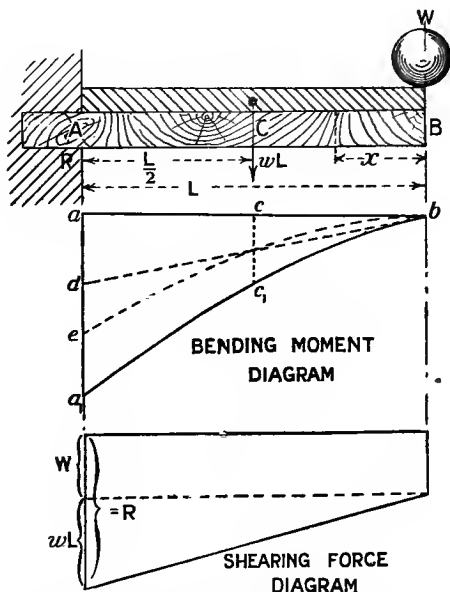
$$\text{S.F.}_x = -(W + wx).$$

The maximum value of the shearing force which occurs at the fixed end of the cantilever A is,—

$$\text{The Maximum S.F.}_A = -(W + wL).$$

To Draw the Diagrams of Bending Moment and Shearing Force.—

(a) You draw the diagram *abd* of bending moment for the concentrated load *W* only.



- (4) CANTILEVER WITH A UNIFORMLY DISTRIBUTED LOAD OVER ITS WHOLE LENGTH TOGETHER WITH A CONCENTRATED LOAD ACTING AT THE FREE END.

(b) Then, draw the diagram *abe* of bending moment for the uniformly distributed load only.

(c) Combine the ordinates of each of the two diagrams *abd* and *abe*, when you obtain the diagram *aba₁*, which represents the combined diagram of bending moments for the concentrated and uniformly distributed loads acting upon the cantilever. In the same way, you obtain the combined diagram of shearing force for both the concentrated and uniformly distributed loads. Conse-

quently, the ordinates of the diagrams of bending moment and shearing force in each case are *equal to the sum* of the ordinates of the diagrams for each load taken separately.

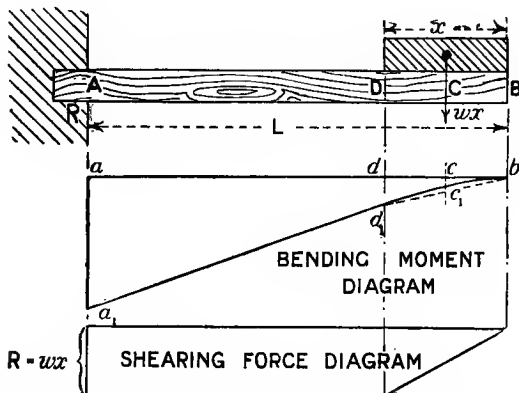
(5) **Cantilever with a Uniformly Distributed Load over Part of its Length.**—Let x be the length of the cantilever AB which is loaded with a uniform load w , per unit of length.

Then, maximum bending moment at the fixed end A is,—

$$\text{The Maximum B.M.}_A = -wx \left(L - \frac{x}{2} \right).$$

The maximum shearing force at the fixed end A is—

$$\text{The Maximum S.F.}_A = -wx = -R.$$



(5) CANTILEVER WITH A UNIFORMLY DISTRIBUTED LOAD OVER PART OF ITS LENGTH.

To Draw the Diagrams of Bending Moments and Shearing Force.—

(a) Let ab represent the cantilever, and draw aa_1 vertically downwards.

(b) Now, consider the load as concentrated at its centre of gravity, and mark off aa_1 to scale, as for a single load wx acting at the point C of the cantilever.

(c) Join a_1 to c the central point of db . It will be found that the vertical ordinate dd_1 represents the value of the bending moment at D, and it is equal to $\frac{wx^2}{2}$.

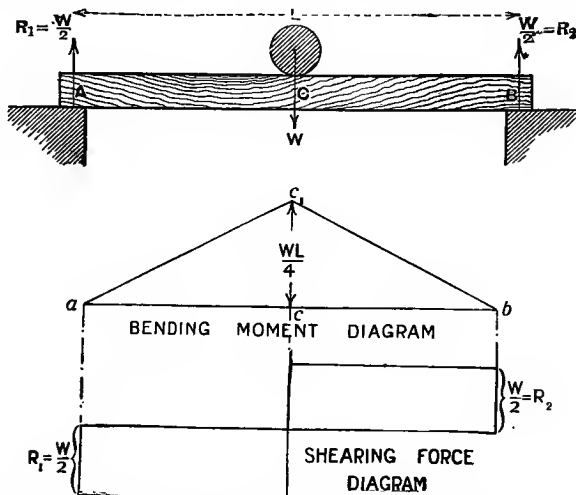
(d) Join d_1 to b .

(e) Now, consider DB as a cantilever fixed at D, and loaded

uniformly along its whole length. Draw the diagram of bending moments *to the same scale*, which will be a *parabola* for this case, so $\frown$.

(f) Take the lengths of several ordinates of this parabola, and at the corresponding points along the inclined base-line d_1b prick off upon the vertical lines these values.

(g) Then abd_1a_1 will be the curve of bending moments for the loaded cantilever, which is made up of the straight line a_1d_1 and



(6) BEAM SUPPORTED AT BOTH ENDS AND LOADED WITH A SINGLE CONCENTRATED LOAD AT THE CENTRE OF THE SPAN.

the semi-parabolic curve d_1c_1b , whilst the straight line is a tangent to the curve at the point d_1 .

The diagram of shearing forces is obtained by similar reasoning.

(6) **Beam Supported at Both Ends and Loaded with a Single Concentrated Load at the Centre of the Span.**—Consider the case of a rectangular beam of span L feet, where the load W is placed at the centre of the beam C , about which we desire to find the transverse stress or bending moment, as well as the shearing force. Then, *neglecting the weight of the beam itself*, and confining our attention solely to the load W , we know that the reactions at A and $B = \frac{W}{2}$.

The bending moment at any section distant x feet from end B of the beam is,—

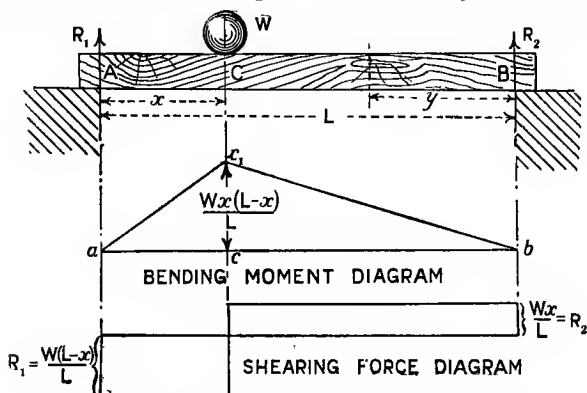
$$\text{B.M.}_x = R_2 x; \text{ and the shearing force } \text{S.F.}_x = R_2 = \frac{W}{2}.$$

The bending moment for any section taken between W and end A is : $\text{B.M.} = R_1 (L - x)$.

The maximum bending moment occurs at the centre of the span, and its value is,—

$$\text{The Maximum B.M.}_0 = R_2 \frac{L}{2} = \frac{WL}{4}.$$

The diagrams of bending moment and shearing force are drawn below the beam, and should present no difficulty to the student.



(7) BEAM SUPPORTED AT BOTH ENDS WITH A SINGLE CONCENTRATED LOAD IN ANY POSITION.

(7) Beam Supported at Both Ends and Loaded with a Single Concentrated Load in Any Position.—Let AB be a uniform beam of length L feet, supported at both ends and carrying a weight W at x feet from the left-hand support A.

Then, by taking moments about the point B, we have—

$$R_1 \times L = W(L - x)$$

$$\therefore \text{reaction at A} = R_1 = \frac{W(L - x)}{L}$$

$$\text{And reaction at B} = R_2 = \frac{Wx}{L}$$

The bending moment at any section of the beam between B and C, and at a distance y feet from B is,—

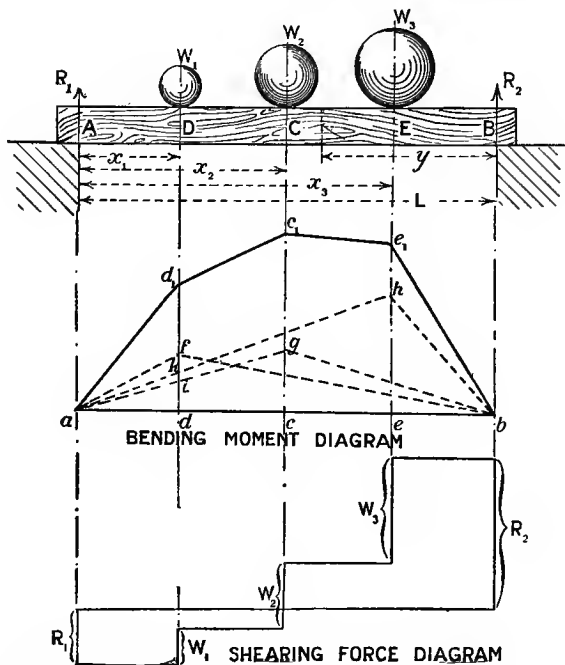
$$\text{B.M.}_y = R_2 y = \frac{Wx}{L} y.$$

The maximum bending moment occurs at the section C of the beam where the load acts, and its value is,—

$$\text{The Maximum B.M.}_C = R_2(L - x) = W \frac{x}{L}(L - x).$$

The bending moment for any section of the beam which may be taken between the points A and C is,—

$$R_2y - W[y - (L - x)] = W \frac{x}{L}y - Wy + W(L - x),$$



(8) BEAM SUPPORTED AT BOTH ENDS AND LOADED WITH SEVERAL SEPARATE CONCENTRATED LOADS.

Or it is—

$$R_1(L - y) = \frac{W(L - x)}{L}(L - y) = W \frac{x}{L}y - Wy + W(L - x).$$

To Draw the Diagrams of Bending Moment and Shearing Force. The vertical ordinate cc_1 represents to scale the value of the bending moment $\frac{Wx(L - x)}{L}$. The bending moment at any other

section of the beam is represented by that portion of the vertical line, which is drawn from the beam, and contained between the horizontal and inclined lines.

In the diagram of shearing forces, the positive shearing force in the right-hand portion of the loaded beam is equal to R_2 , whilst the negative shearing force in the left-hand portion is equal to R_1 or $R_2 - W$. The two rectangles which make up the diagram of shearing force have equal areas.

(8) **Beam Supported at Both Ends and Loaded with Several Separate Concentrated loads.**—Let L be the length of the span, and W_1, W_2, W_3 , the loads which are acting at the respective distances x_1, x_2, x_3 from the left-hand support in the figure on the previous page.

Then,

$$\text{Reaction at B} = R_2 = \frac{W_1x_1 + W_2x_2 + W_3x_3}{L}.$$

$$\text{Reaction at A} = R_1 = \frac{W_1(L - x_1) + W_2(L - x_2) + W_3(L - x_3)}{L}.$$

To find the bending moment at any section distant y feet from the right-hand support, say between E and C , let us consider the right-hand portion, when we get

$$\text{B.M.}_y = R_2y - W_3[y - (L - x_3)].$$

To Draw the Diagrams of Bending Moment and Shearing Force. The diagram of bending moments may readily be obtained by drawing the diagrams for each concentrated load separately in the first instance, and then combining them by adding the ordinates as shown in the figure, where $dd_1 = dl + dk + df$. The separate diagrams are shown in dotted lines, and the final diagram, derived from them, is shown in full black lines.

The shearing force diagram is shown and is obtained by superposition in a similar manner to the bending moment diagram.

(9) **Beam Supported at Both Ends, and Loaded Throughout its Length with a Uniformly Distributed Load.**—Let L be the length of the beam, and the uniform load be w units per foot run of span. Then, the total weight $W = wL$. The reaction at each support is $\frac{wL}{2}$.

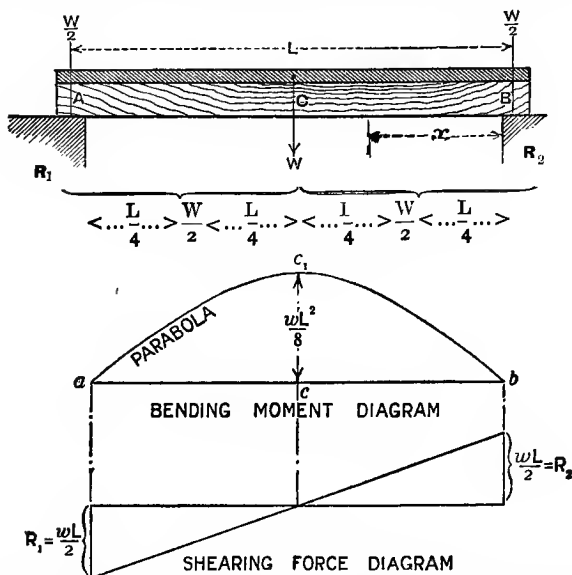
The bending moment at any section distant x from the right-hand end B of the beam is,—

$$\text{B.M.}_x = (R_2x - wx \times \frac{x}{2}) = \frac{w}{2} (Lx - x^2) = \frac{wx}{2} (L - x).$$

This is a maximum when $x = \frac{1}{2}L$,

$$\therefore \text{The Maximum B.M.}_c = \frac{wL^2}{8} = \frac{WL}{8}.$$

The curve of bending moments is a parabola with its origin of co-ordinates at a point on the curve distant $\frac{L}{2}$ from the axis.



(9) BEAM SUPPORTED AT BOTH ENDS, AND LOADED THROUGHOUT ITS LENGTH WITH A UNIFORMLY DISTRIBUTED LOAD.

Hence, to draw the diagram of bending moments, you set up the ordinate cc_1 from the centre, to represent $\frac{wL^2}{8}$, and draw a parabola passing through ac_1b .

The shearing force at any section distant x feet from the right-hand support is,—

$$\text{S.F.}_x = R_2 - wx = w\left(\frac{L}{2} - x\right).$$

The shearing force is positive when x is less than $\frac{L}{2}$, and negative when x is greater than $\frac{L}{2}$. The shearing force gradually

decreases from the value $\left(-\frac{wL}{2}\right)$ at the left hand support, to zero at the middle, and increases uniformly to $\left(+\frac{wL}{2}\right)$ at the right-hand support.

EXAMPLE II.—A uniform beam 12 feet long weighs 400 lbs., and is supported at its extremities. Find the bending moment tending to break the beam at a point 3 feet from one end, and the shearing force.

ANSWER.—Here we have only to consider the weight of the beam just as if we had been considering a uniformly distributed load. Consequently, the previous figure will help the student, for the point about which we have to take moments is evidently

3 feet from one end, or $\frac{3'}{12} = \frac{1}{4}$ of the whole length between the supports from one end.* Let that point be $\frac{L}{4}$ from A. Then the weight of this part $\frac{L}{4} = \frac{1}{4}$ of 400 lbs. = 100 lbs., and may be considered to act at a point $\frac{L}{8}$ from A, or halfway along $\frac{L}{4}$ from A and *downwards*. In the question $\frac{W}{4} = 100$ lbs. for the whole weight of the beam, or $wL = W = 400$ lbs. The total weight W produces *upward* reactions at A and at B = $\frac{W}{2} = 200$ lbs.

Then, by taking moments about the point 3 feet from A. we have,—

$$\text{B.M.} = \left(\frac{W}{2} \times \frac{L}{4} - \frac{W}{4} \times \frac{L}{8}\right) = \left(\frac{WL}{8} - \frac{WL}{32}\right) = \frac{3}{32}WL = \frac{3 \times 400 \times 12}{32} = 450 \text{ lbs.-ft.}$$

N.B.—Students may always check the B.M. result as found from one end, by taking moments about the same point from the other end. The two results *must* be equal to each other, for there is equilibrium between their effects. Therefore, by taking moments about the same point 9 feet from B we have,—

* If the student experiences any difficulty in understanding the above reasoning, he should at once draw down a figure to scale, marking all distances, weights, reactions, &c., at their proper places.

$$B.M. = \left(\frac{W}{2} \times \frac{3}{4}L - \frac{3}{4}W \times \frac{1}{2} \times \frac{3}{4}L \right)$$

$$B.M. = \left(\frac{3WL}{8} - \frac{9WL}{32} \right) = \left(\frac{12WL - 9WL}{32} \right) = \frac{3WL}{32}$$

$$B.M. = \frac{3 \times 400 \times 1}{32} = 450 \text{ lbs.-ft.} \text{ Or, the same result as before.}$$

Shearing Force.—As was previously pointed out in this Lecture, the shearing force or load at *any* point or *any* transverse section of the beam is equal to the resultant or algebraical sum of all the parallel forces on *either* side of the point or section.

Consequently, in this example, the forces on the A side of the section, where the shearing force is asked for, are $\frac{W}{2}$ acting vertically upwards at A and $\frac{W}{4}$ downwards.

∴ *The Shearing Force to Left of the Section*

$$= \frac{W}{2} - \frac{W}{4} = \frac{W}{4} = \frac{400}{4} = 100 \text{ lbs. upwards.}$$

The Shearing Force to Right of the Section

$$= \frac{3W}{4} - \frac{W}{2} = \frac{W}{4} = \frac{400}{4} = 100 \text{ lbs. downwards.}$$

(10) **Beam Carrying a Uniformly Distributed Load over the Whole of its Length and a Single Concentrated Load.** Let AB represent the beam of length L feet supported at A and B. This beam is carrying a uniformly distributed load of w units per foot-run of span, together with a concentrated load W acting at the point D as represented by the figure on next page.

$$\text{The reaction at B} = R_2 = \frac{wL}{2} + \frac{W}{4}.$$

The shearing force at any section between B and D distant x feet from B is,—

$$S.F._x = R_2 - wx = \frac{wL}{2} + \frac{W}{4} - wx.$$

The shearing force at support B is,—

$$S.F._B = R_2 = \frac{wL}{2} + \frac{W}{4}.$$

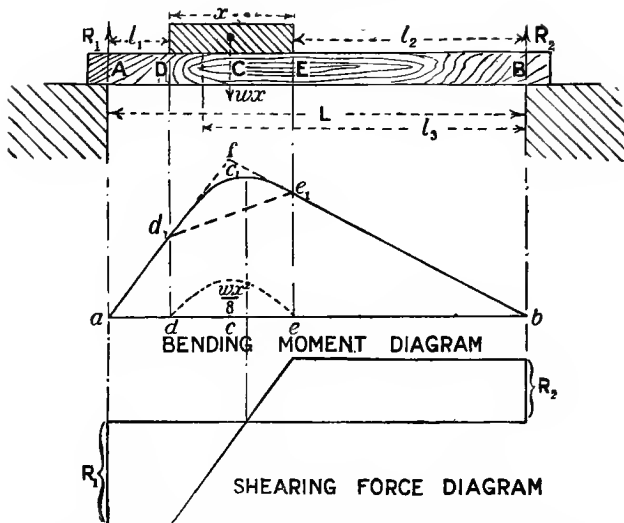
The shearing force at section D is,—

$$S.F._D = R_1 - w \frac{3L}{4}.$$

The reactions at the supports are

$$R_1 = wx \frac{l_2 + \frac{x}{2}}{L}$$

$$R_2 = wx \frac{L - (l_2 + \frac{x}{2})}{L} = \frac{wx(l_1 + \frac{x}{2})}{L}$$



(II) BEAM SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY OVER A PORTION OF ITS LENGTH NOT EXTENDING TO EITHER ABUTMENT.

The bending moment at any section *between B and E*, where l_3 is the distance from the right-hand abutment, is given by the equation,—

$$\text{B.M.}_{l_3} = R_2 l_3 = \frac{wx l_3}{L} \left(l_1 + \frac{x}{2} \right).$$

The bending moment for any section *between E and D*

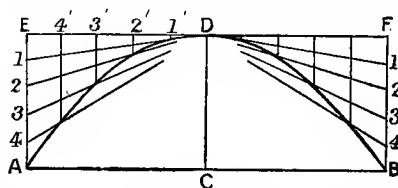
$$\text{B.M.}_{l_3} = R_2 l_3 - w \frac{(l_3 - l_2)^2}{2} = \frac{wx l_3}{L} \left(l_1 + \frac{x}{2} \right) - \frac{w(l_3 - l_2)^2}{2}.$$

The bending moment for any section *between D and A*

$$\text{B.M.}_{l_3} = R_1 (L - l_3) = R_2 l_3 - wx \left\{ l_3 - \left(l_2 + \frac{x}{2} \right) \right\}.$$

From one of these three equations we can determine the bending moment at any section of the beam.

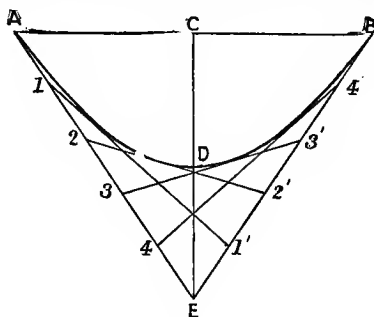
To Draw the Diagram of Bending Moments.—Let ab represent the beam. Then, the load extending over the portion de (which is equal to x) acts at C , which is the centre of DE . Consider the weight of the whole load as if it were concentrated at C . Draw the diagram afb for this load. Now consider the length DE as if it were a uniformly loaded beam supported at D and E . Draw the parabola on de for this load (to the same scale). From d and e draw verticals dd_1 and ee_1 meeting af and bf at d_1 and e_1



respectively. Join d_1e_1 and draw vertical ordinates above d_1e_1 equal to the ordinates of parabola on de . Then $ad_1c_1e_1b$ is the diagram of bending moments.

Graphic Construction of the Parabola upon a Given Base, while the Height is Obtained by Calculation, in the Case of a Beam Carrying a Uniformly Distributed Load.*

First Method.—Let AB be the given base or length of beam, and



let D be the vertex of the required curve, which is directly over C , the middle point of AB . The parabola in this instance will be symmetrical.

* See *The Trans. of the Inst. of Engs. and Shipbuilders*, vol. xxxiii., 1889-90, for paper on "The Alexander-Thomson Moment Delineator, and its Application to Maximum Bending Moments due to Moving Loads," by Prof. Thomas Alexander, M.A., and Mr. Arthur W. Thomson, B.Sc.

Set off CD perpendicular to AB, and make its height equal to the maximum bending moment. Complete the rectangle AEFL. Divide AE into any number of equal parts, and ED into the same number, marking each division as shown by the accompanying figure. Join the vertex D to the points 1, 2, 3, 4, and through the points 1', 2', 3', and 4' draw lines parallel to DC cutting the lines drawn through D. The points of intersection of corresponding pairs of lines are points on the curve. Then the curve is drawn from A, through the points of intersection to D. The figure being symmetrical, the other side will be the same, and when completed, will give a ready approximation to the *parabola*.

Second method.—Suppose, as in the first method, that we found by calculation the maximum bending moment. Then take C, the middle point of AB, and draw CE perpendicular to AB, setting off CD and DE each equal to the maximum bending moment. Join EA and EB, and divide these lines into any, but the same, number of equal parts at 1, 2, 3, 4, and 1', 2', 3', 4'. Join 1, 1'; 2, 2'; 3, 3'; 4, 4'. These lines will be tangents to the required parabolic curve, and can be drawn to touch these lines.

LECTURE VI.—QUESTIONS.

1. A beam of wood, rectangular in section, is fixed at one end and loaded at the other. What is occurring at various places in any imaginary cross section? Sketch any thing you have seen or used which illustrates your ideas about bending.

2. State clearly what occurs at a cross section of a loaded beam. What is the neutral line of the section? Where is it? (B. of E. 1904.)

3. A uniform beam 10 feet long, and weighing 1000 lbs., is supported at both ends. A weight of 100 lbs. is placed at a distance of two feet from one end; find the pressure and reaction at each point of support and make a side view of the arrangement to scale, marking on your sketch the weights, distances, and reactions at each place. *Ans.* 580 lbs.; 520 lbs.

4. A cantilever projects 5 feet from a wall and carries a load of 400 lbs. at its free end. Calculate the bending moment and shearing force at (i.) the wall, (ii.) its middle.

5. A cantilever, 16 feet long, supports four loads of 3, 5, 6, 8 tons, situated at distances from the fixed end of 16, 12, 6, 3 feet respectively. Find the bending moments at the fixed end, and at a section 9 feet from the fixed end.

6. A cantilever, 6 feet long, carries a load of 600 lbs. uniformly distributed. Calculate the bending moment and shearing force at intervals of 2 feet along its length.

7. A cantilever, 4 ft. long, carries a uniform load of 5 cwt. per foot run. and a load of 1 ton at its extremity. Sketch the curves of shearing force and bending moment, and find their values at 2 and 4 ft. from the free end of the cantilever.

Ans. S.F. at 2 ft. from free end = 1·5 tons.

B.M. at " " " = 2·5 tons-ft.

S.F. at 4 ft. " " = 2 tons.

B.M. at " " " = 6 tons-ft.

(C. & G., 1903, O. Sec. B.)

8. A cantilever, 20 feet long, has a load of 1 ton per foot-run distributed over one-half of the beam reckoning from the free end. Determine the bending moments and shearing forces at the fixed end, and at 6, 8, 12, and 18 feet from the fixed end.

9. A beam 12 feet long is supported at its ends, and is loaded with a weight of 3 tons at a point two feet from one end. Find the bending moment at the centre of the beam, and also the shearing force.

Ans. B.M. = 36 ton-inches, S.F. = 0·5 ton.

10. A beam 40 feet long is supported at its ends, and is loaded with weights of 5, 6, 7, 8 tons, situated at points distant from the left abutment of 5, 10, 20, and 30 feet respectively. Draw the diagrams of bending moment and shearing force, and find the values at these points.

Ans.

11. A beam 30 feet long is supported at both ends, and is loaded throughout its length with a uniform load of $1\frac{1}{2}$ tons per foot run. Draw diagrams of bending moment and shearing force, and find the values at $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{2}$ the span from the left abutment.

12. A girder, which rests on supports at its end, is 10 ft. long, and carries a uniformly distributed load of 1 ton per foot run over 5 ft. of its length, the load commencing $1\frac{1}{2}$ ft. from the left-hand support. Calculate the shearing force and bending moment at a distance of 3 ft. from the left-hand support. *Ans.* S.F. = -1·5 tons; B.M. = 7·875 tons-feet.

(O. & G., 1904, O. Sec. B.)

13. A beam 60 feet span is loaded uniformly with 1 ton per foot-run over the portion commencing at 12 feet from the left support and ending at the middle of the beam, or for a distance of 18 feet. Find the bending moments at each end of the load, and also the position and magnitude of the maximum bending moment. Draw the diagrams of bending moments and shearing force.

14. A beam 50 feet span is loaded uniformly for a distance of 36 feet, measured from the left abutment, with 10 cwts. per foot-run. Find the position of the maximum bending moment, and its amount; also, find the bending moment at the centre of the beam, and draw the diagrams of bending moment and shearing force.

15. Define what is meant by the bending moment and the axis of a beam. A uniform beam 10 feet long weighs 500 lbs., and is supported at its extremities. Find the bending moment tending to break it at a point 4 feet from one end. *Ans.* 600 lbs.-ft.

16. A bar of pine 44 inches long rests on props at its extremities, and just supports 10 weights, of 14 lbs. each, hung at equal intervals of 4 inches along the rod. Find the amount of a single weight which, if hung at the centre of the bar, would strain it to the same extent. *Ans.* 76.36 lbs.

17. A batten of fir, 6 feet in length and supported at its extremities, will just sustain a load of 520 lbs. when hung at the centre. If this weight be removed, and two weights, each equal to P lbs., be hung at distances of 2 and 4 feet along the bar, what is the greatest value which may be assigned to P ? *Ans.* 390 lbs.

18. A strip of pine, 30 inches in length between the points of support, is loaded with 26 lbs. hung at the centre. If weights of W lbs. are hung on the strip at distances of 10 and 20 inches from one end, what is the value of W which will produce the same bending moment at the centre as in the first instance? *Ans.* $W = 19\frac{1}{2}$ lbs.

N.B.—See *Appendices B and C* for other questions and answers.

LECTURE VII

CONTENTS.—Beam hinged at one end and overhanging a pier with Loading at the Free end and between the supports—Beam hinged at one end and projecting as a cantilever beyond its other support, with a uniform load upon the beam and a single concentrated load at the projecting end—Beam supported at two points intermediate between the ends, and loaded at its extremities and centre, with concentrated weights—Beam overhanging two Piers, loaded with a single concentrated load at each end, and a uniform load between the piers—Beam overhanging two Piers, and loaded with a uniformly distributed load over its whole length—Fixed or Encastré Beams—Beam of Uniform section securely fixed at the Ends and loaded at the centre—Beam of uniform section fixed horizontally at the ends and uniformly loaded—Beam of uniform section fixed at both ends and loaded uniformly over a portion of its length, but not extending to either abutment—Moment of Resistance opposed to the Bending Moment—Strength of Rectangular Beams—Relative Strength of Rectangular Cantilevers and Beams supported and loaded in Different Ways—Comparison of the Loads and Sizes of Beams by Proportion—Example I.—Different Sections of Cast-Iron, Wrought-Iron, and Steel Beams—Cast-Iron Beams or Girders—Wrought-Iron Beams and Girders—Steel Beams—Questions.

(12) Beam Hinged at one end, and overhanging a Pier, with loading at the free end and between the supports.—Let the beam AD be anchored down or hinged to the abutment A, and resting on the pier B. Let the beam be loaded with concentrated weights at C and D.

The reaction R_2 is obtained by taking moments about the hinged end A, thus,—

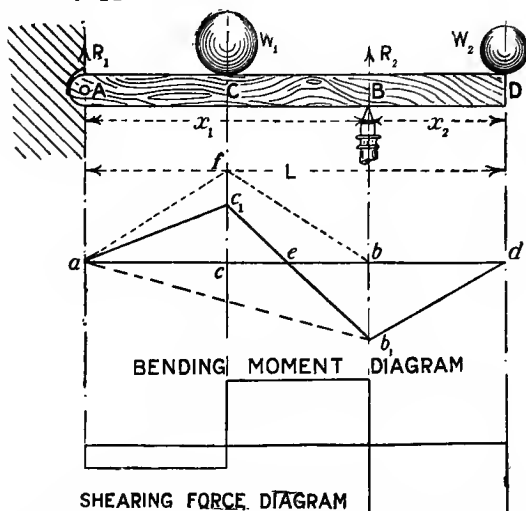
$$R_2 \times BA = W_1 \times CA + W_2 \times DA.$$

$$\therefore R_2 = \frac{W_1 \times CA + W_2 \times DA}{BA}.$$

Knowing the reaction R_2 , we can find the bending moment at any section of the length between A and B.

The bending moment diagram ac_1b_1d is obtained by drawing ab_1d , which is the diagram due to the load W_2 at D alone; also, afb which is due to the load W_1 at C alone and superposing them. Or, more directly, we obtain ac_1b_1 by setting up from the sloping base ab_1 the ordinates of the diagram due to the load at C.

To draw the shearing force diagram, we find the reaction at B, which is the step by which the shearing force changes there, and then draw the diagram from left to right. Or alternatively, sketch the diagram for the load at D alone, and the reaction at B due to that load being $W_2 \frac{AD}{AC}$. Then, superpose the diagram for the load W_1 applied at C.



(12) BEAM HINGED AT ONE END AND OVERHANGING A PIER WITH LOADING AT THE FREE END AND BETWEEN THE SUPPORTS.

If, as in the diagram, c_1 is above the horizontal line ad , the beam is "sagging" from a to e , and "hogging" from e to d .

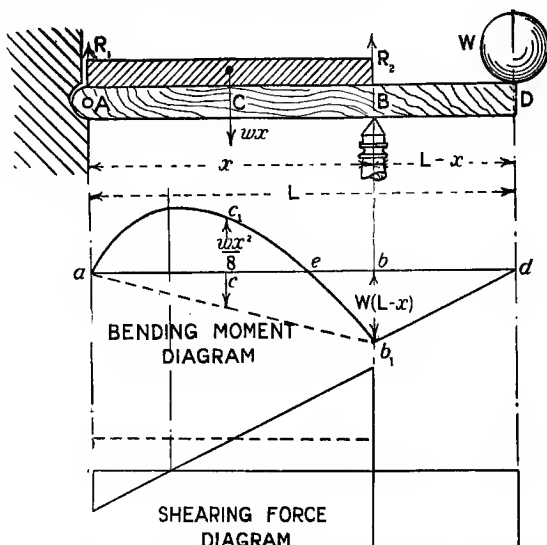
The point e where the curve changes is called the point of *contrary flexure*.

(13) Beam hinged at one end, and projecting as a cantilever beyond its other support, with a uniform load upon the beam, and a single concentrated load at the projecting end.—The bending moment diagram db_1a for the single concentrated load W is first drawn, and then the B.M. diagram for the distributed load is drawn on the sloping base b_1a . Similarly draw the shearing force diagram for the single concentrated load W , and superpose on it the shearing force diagram for the distributed load, when we get the final diagram of shearing forces for the combined loads acting on the beam.

(14) Beam supported at two points intermediate between the ends.

Let ED represent the beam supported at A and B, where $EA = x_1$, $AB = x_2$, $BD = x_3$.

(a) First consider the case when such a beam is loaded at its extremities with two weights W_1 and W_3 .



(13) BEAM HINGED AT ONE END AND PROJECTING AS A CANTILEVER BEYOND ITS OTHER SUPPORT, WITH A UNIFORM LOAD UPON THE BEAM AND A SINGLE CONCENTRATED LOAD AT THE PROJECTING END.

The Reactions or values of the supporting forces at A and B are,—

$$R_1 = \frac{W_1(x_1 + x_2) - W_3x_3}{x_2}, \text{ by taking moments about the support B.}$$

$$R_2 = \frac{W_3(x_2 + x_3) - W_1x_1}{x_2}, \text{ by taking moments about the support A.}$$

The Bending moments at A and B are—

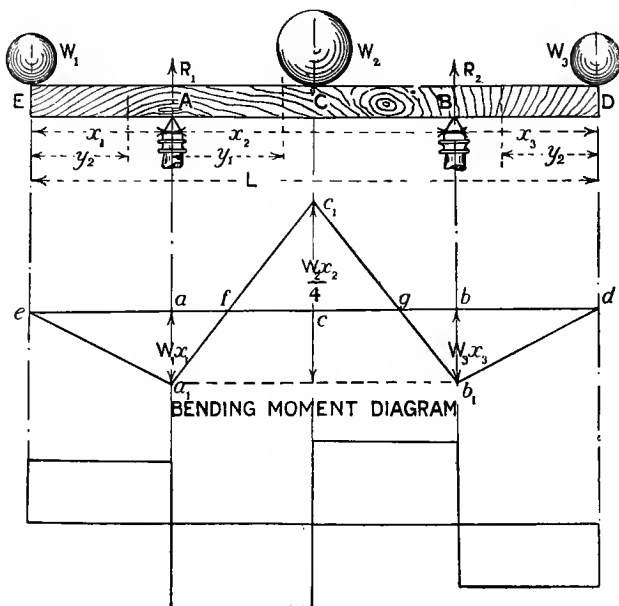
$$- \text{B.M.}_A = W_1x_1 \text{ or } = W_3(x_2 + x_3) - R_2x_2.$$

$$- \text{B.M.}_B = W_3x_3 \text{ or } = W_1(x_1 + x_2) - R_1x_2.$$

The bending moment at any section of the beam situated at a distance from the support $A = y_1$, is,—

$$- \mathbf{B.M.}_{y_1} = W_1(x_1 + y_1) - R_1 y_1 = W_3(x_2 + x_3 - y_1) - R_2(x_2 - y_1).$$

(b) Consider the case of a beam similar to the above, but



SHEARING FORCE DIAGRAM

(14) BEAM SUPPORTED AT TWO POINTS INTERMEDIATE BETWEEN THE ENDS.

loaded with an additional concentrated weight W_2 at the middle point between A and B , as shown by the accompanying diagram. Then, by taking moments about the support B .

$$R_1 = \frac{W_1(x_1 + x_2) + W_2 \frac{x_2}{2} - W_3 x_3}{x_2},$$

And by taking moments about the support A

$$R_2 = \frac{W_2 \frac{x_2}{2} + W_3(x_2 + x_3) - W_1 x_1}{x_2},$$

The bending moments at A, B, and C are,—

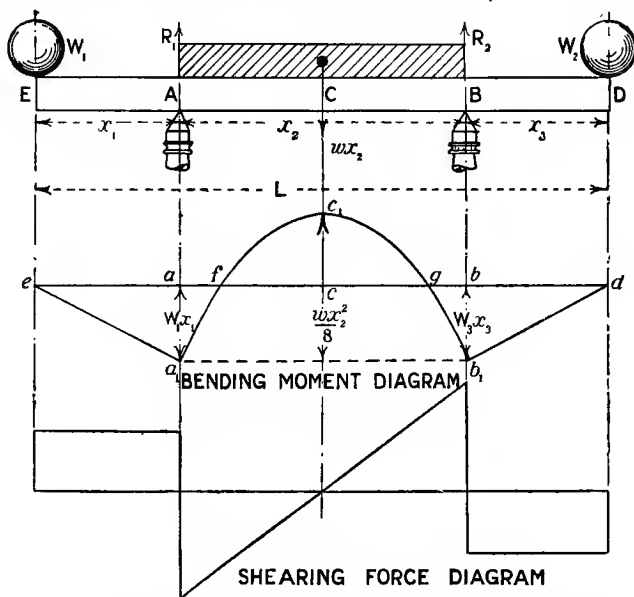
$$- \text{B.M.}_A = W_1 x_1, \text{ or } = W_3(x_2 + x_3) + W_2 \frac{x_2}{2} - R_2 x_2.$$

$$- \text{B.M.}_B = W_3 x_3, \text{ or } = W_1(x_1 + x_2) + W_2 \frac{x_2}{2} - R_1 x_2.$$

$$\pm \text{B.M.}_C = R_1 \frac{x_2}{2} - W_1 \left(x_1 + \frac{x_2}{2} \right), \text{ or } = R_2 \frac{x_2}{2} - W_3 \left(\frac{x_2}{2} + x_3 \right).$$

The value of the bending moment at any section of the projecting arms EA or DB is given by the equation,—

$\text{B.M.}_{y_2} = W_1 y_2$, or $-\text{B.M.}_{y_2} = W_3 y_2$, where y_2 = distance of the section from the end of the beam.



- (15) BEAM OVERHANGING TWO PIERS. LOADED WITH A SINGLE CONCENTRATED LOAD AT EACH END, AND A UNIFORM LOAD OF w UNITS PER FOOT-RUN BETWEEN THE PIERS.

Also, the general expression for the bending moment of any section of the beam between the supports A and B, at a distance y_1 from A, is,—

$$\pm B.M._{y_1} = R_1 y_1 - W_1(x_1 + y_1),$$

$$\text{or } \pm B.M._{y_1} = R_2(x_2 - y_1) - W_2\left(\frac{x_2}{2} - y_1\right) - W_3(x_2 + x_3 - y_1),$$

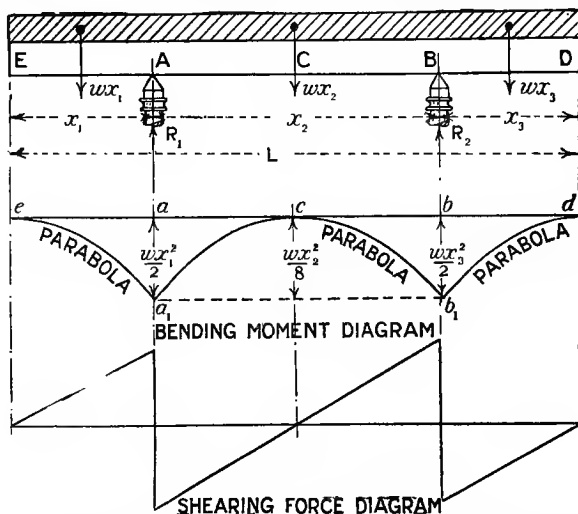
when the section lies to the left of W_2 . And,—

$$\pm B.M._{y_1} = R_1 y_1 - W_2\left(y_1 - \frac{x_2}{2}\right) - W_1(x_1 + y_1),$$

$$\text{or } \pm B.M._{y_1} = R_2(x_2 - y_1) - W_3(x_2 + x_3 - y_1),$$

when the section lies to the right of W_2 .

It will be seen from the diagram of bending moments, that $B.M._{y_1}$ may be either positive or negative according to the relative values of the different weights and linear dimensions. It will also be noticed from the diagram of bending moments, that the



(16) BEAM OVERHANGING TWO PIERS AND LOADED WITH A UNIFORMLY DISTRIBUTED LOAD OF w UNITS PER FOOT-RUN OVER ITS WHOLE LENGTH.

values at the supports A and B are not affected by the addition of the central weight W_2 .

The diagram of shearing forces will present no difficulty when drawing it, if the student will consider for a moment, the three sections into which the beam may be sub-divided, and the respective diagrams for each part as separately loaded.

(15) Beam overhanging two Piers. Loaded with a single concentrated load at each end, and a uniform load of w units per foot-run between the Piers.—It will be noticed on

comparing this set of figures with the last set, that the *only* difference between the diagrams of bending moments and shearing forces lies in the part contained between the supports A and B. This difference is at once apparent as being due to the nature of the loading in each case. Consequently, we must superpose upon the diagram of bending moments between the points a_1, b_1 , a "*parabolic curve*" for the uniformly loaded part AB of the beam ED.

Also, in the shearing force diagram, we substitute the shearing force diagram for a uniformly loaded beam between the points of support A and B.

(16) **Beam overhanging two Piers, and loaded with a uniformly distributed load of w units per foot-run over its whole length.**—The following figures will enable the student to understand this variation of the previous case without further detailed explanation, and thus encourage him when mastered to correctly interpret such diagrams as well as plot them out for other similar cases.

Fixed or Encastré Beams.—An encastré or built-in beam is one whose ends are secured tangentially—i.e., horizontally at right angles to the supporting wall—in such a way, that the fixed ends remain horizontal when the beam bends under the load. Thus the two end portions are in "*hogging*" or being curved upwards, whilst the central portion is "*sagging*" or curved downwards. The points where the curvature alters from *hogging* to *sagging* are called the "*points of contrary flexure*" or "*points of inflection*," and with beams of uniform section there is theoretically no bending moment at the two points. The beam may therefore be considered as being made up of two cantilevers (one at each end), and a central beam supported on the ends of those cantilevers.

(17) **Beam of uniform section securely fixed at the ends and loaded at the centre.**—It can be proved by mathematical investigation that with a beam AB fixed and loaded with a central weight W , the points of inflection are at a distance $\frac{L}{4}$ from each abutment, where L = the length of the beam.

First. Consider the central portion DE, which may be taken as an independent beam of span $\frac{L}{2}$, resting on two supports at D and E, and loaded with a central weight W . The vertical reaction at each end is equal to $\frac{W}{2}$.

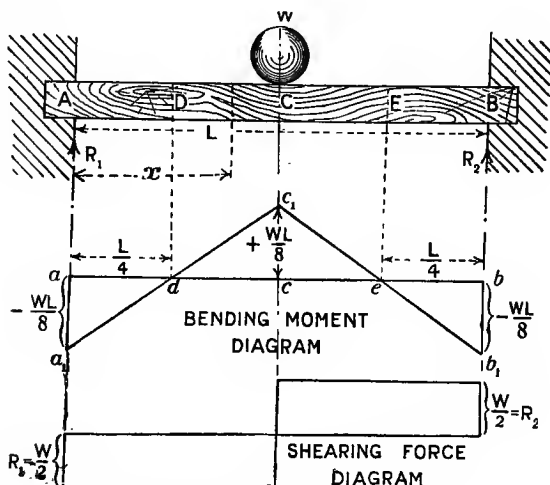
The bending moment at the centre is—

$$\text{The maximum B.M.}_c = \frac{WL}{8}.$$

And the bending moment at a section distant x feet from the abutment is,—

$$\text{B.M.}_x = -\frac{W}{2} \left(x - \frac{L}{4} \right), \text{ when } x > \frac{L}{4}.$$

Second. Consider the end portions AD and BE. These are equivalent to cantilevers fixed at the ends A and B, and loaded at the free ends with weights $\frac{W}{2}$.



(17) BEAM OF UNIFORM SECTION SECURELY FIXED AT THE ENDS AND LOADED AT THE CENTRE.

The maximum bending moments of these cantilevers occur at the fixed ends, and the values are found as follows—

$$\text{The maximum B.M.}_A = \text{B.M.}_B = -\frac{WL}{8}.$$

The bending moments at the centre and at the two ends are equal to each other, but they are of opposite sign, the bending moment at the centre being positive, and the bending moments at the abutments negative.

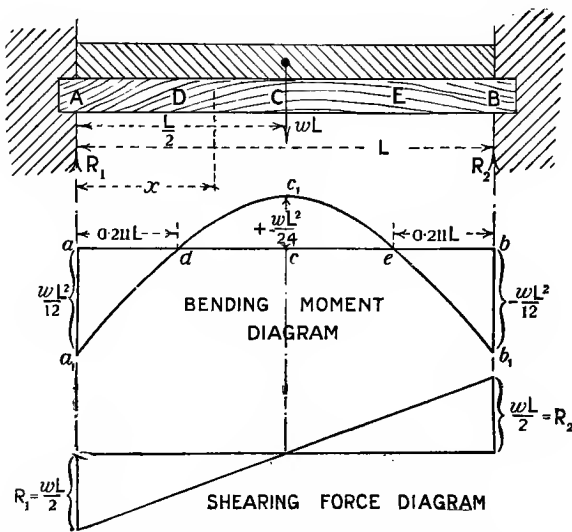
The diagrams of bending moment and shearing force have the forms outlined in the figures. Also, it will be observed, from a comparison of the diagrams of bending moments, that a beam of uniform section with its ends *firmly fixed* is *twice as strong in*

resisting a transverse stress as a beam of the same length and section, *whose ends are free*, because, the maximum bending moment in the first case is only one-half that in the second case.

(18) **Beam of uniform section fixed horizontally at the ends and uniformly loaded.**—AB represents the beam of length L with its ends firmly fixed at the abutments, and uniformly loaded with w units per foot-run of length.

The reactions at the abutments are each equal to $\frac{wL}{2}$.

A uniformly loaded beam is bent in a similar manner to that in



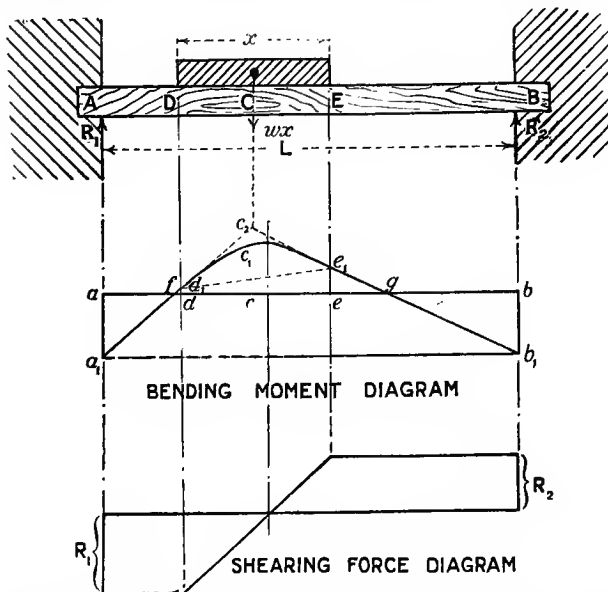
(18) BEAM OF UNIFORM SECTION FIXED HORIZONTALLY AT THE ENDS AND UNIFORMLY LOADED.

the previous case, but the *points of contrary flexure* d and e do not occur at the same positions. It can be proved by mathematical analysis, that in fixed beams which are uniformly loaded, the *points of contrary flexure* occupy the following positions along the beam : $ad = be = .211L$, and $de = .578L$.

The beam in this case, may also be resolved into three independent ones, viz.—

(i) A central beam de of length $= .578L$, which rests on two supports at d , e , and loaded uniformly over its length with w units per foot-run ; as well as,—

(ii.) Two cantilevers ad , be of length = $\cdot 211L$, which are loaded with a uniform load of w units per foot-run, together with a concentrated load acting at their extremities = $\cdot 289wL$ units



(19) BEAM OF UNIFORM SECTION AND FIXED HORIZONTALLY AT ITS ENDS WITH A UNIFORM LOAD OVER PART OF ITS LENGTH.

The maximum bending moment at the centre C is,—

$$\text{The maximum B.M.}_C = \frac{w(\cdot 578L)^2}{8} = \frac{wL^2}{24}.$$

$$\text{The maximum B.M.}_A = \text{B.M.}_B$$

$$\text{The maximum B.M.}_A = \frac{w(\cdot 211L)^2}{2} + \cdot 289wL \times \cdot 211L$$

$$\text{The maximum B.M.}_A = -\frac{wL^2}{12}$$

The bending moment at any section of the loaded beam distant x feet from the abutment is—

$$\text{B.M.}_x = \frac{wx}{2}(L-x) - \frac{wL^3}{12}.$$

From the above maximum values of the bending moments, it will be seen, that the bending moment at the centre is *one-third* of the

value for the same beam, which is uniformly loaded and *free* at the ends. Also, that the bending moments at the abutments are *double* the value at the centre of the beam. Consequently from these observations it follows, that a beam which is firmly fixed at the ends is one and a half times as strong as the same beam when used with its ends merely resting on the abutments.

To draw the diagram of bending moments.—Let ab represent the beam, and make $ad = be = \cdot 211L$. Draw the vertical cc_1 , and make it equal to $\frac{wL^2}{24}$ to scale. Then, draw the parabolic curve ec_1d . Now, draw the verticals aa_1 , bb_1 downwards, and equal by scale to $\frac{wL^2}{12}$. Finally, construct the curves da_1 and eb_1 .

(19) Beam of uniform section, and fixed horizontally at its ends with a uniform load over part of its length.—The previous set of figures will enable the student to understand this variation of the previous case.

Moment of Resistance opposed to the Bending Moment.

—It can be proved by mathematics, that the *resultant of all the tensional stresses* (on one side of the neutral axis of any transverse or cross section of a beam) *is equal to the resultant of all the compressive stresses* (on the other side of the neutral axis at the particular cross section considered). These two equal and opposite forces constitute a couple, whose moment is opposite in direction and equal in magnitude to the *bending moment* at the cross sections. It therefore constitutes the *moment of resistance* of the beam, which is opposed to the *bending moment*.

Strength of Rectangular Beams.—The *resisting moment* of a *cantilever* of rectangular cross section, loaded at the outer end, as illustrated by the first figure in this Lecture, is expressed by the formula—

$$\text{R.M.} = kBD^3$$

Where R.M. = Resisting moment in inch lbs.

“ k = Constant number found by trial depending on the nature of the material of which the beam is composed.

“ B = Breadth of beam in inches.

“ D = Depth of beam in inches.

Then if W = Weight or load tending to bend or break beam in lbs

And if L = Length of beam in inches.

The *Bending Moment* = The *Resisting Moment*.

Or, ... B.M. = R.M.

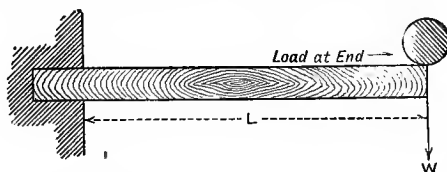
i.e., ... $W \times L = k \times B \times D^3$

∴ $W = k \frac{BD^3}{L}$

Hence the general rule, the *strength of rectangular beams to resist bending is directly proportional to the breadth, to the square of the depth, and inversely proportional to the length of the beam.*

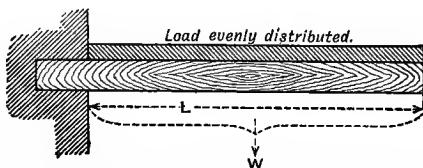
Relative Strength of Rectangular Cantilevers and Beams supported and loaded in Different Ways.—We have already proved the relative values of the bending moments for rectangular beams supported and loaded in the following ways, and we have also proved that the relative bending moments are inversely as the relative strengths or loads which they can support. Hence for cantilevers and beams :

ILLUSTRATIONS, EXPLANATIONS, AND FORMULÆ FOR RECTANGULAR BEAMS SUPPORTED IN DIFFERENT WAYS.



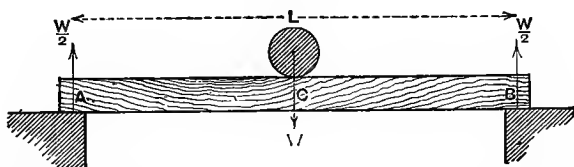
CASE I.—Fixed at one end and loaded at the other. The bending moment has maximum advantage. Therefore—

$$W = k \frac{BD^2}{L}$$



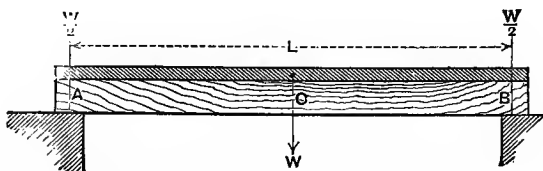
CASE II.—Fixed at one end and loaded uniformly. Here the bending moment has only $\frac{1}{2}$ the advantage that it has in Case I. Therefore—

$$W = 2k \frac{BD^2}{L}$$



CASE III.—Supported at both ends and loaded at the centre. Here the bending moment has only $\frac{1}{4}$ the advantage that it has in Case I. Therefore—

$$W = 4k \frac{BD^2}{L}$$



CASE IV.—Supported at both ends and loaded uniformly. Here the bending moment has only $\frac{1}{8}$ the advantage that it has in Case I. Therefore—

$$W = 8k \frac{BD^2}{L}$$

WAYS IN WHICH BEAMS ARE SUPPORTED AND LOADED.	Relative B.Ms.	Relative B.Ms. or Strengths.	Relative Stiffness.	Relative Deflections.
I. Fixed at one end and loaded at the other ..	1	1	1	128
II. Fixed at one end and loaded uniformly .	$\frac{1}{2}$	2	$2\frac{1}{2}$	48
III. Supported at both ends and loaded at centre .	$\frac{1}{4}$	4	16	8
IV. Supported at both ends and loaded uniformly	$\frac{1}{8}$	8	25.6	5
V. Fixed at both ends and loaded at centre .	$\frac{1}{8}$	8	64	2
VI. Fixed at both ends and loaded uniformly .	$\frac{1}{12}$	12	128	1

Comparison of the Loads and Sizes of Beams by Proportion.—We have already stated that the constant numerical value represented by k (in the preceding formula for the strengths of beams having a rectangular cross section) has to be ascertained by trial. The usual way is to take a comparatively small beam of the same material, and to support as well as load it in the precise manner that the actual beam has to be supported and loaded.

Then, by carefully ascertaining *either* the *breaking load* of this elementary beam (if it should be the *ultimate strength* that is required), *or* the load which produces a certain safe *ratio* of the deflection from the horizontal *to the distance* of the load from its support (if it should be the *safe load* that is desired), you substitute the numerical values of these loads in the formula, and thereby ascertain the probable ultimate strength or safe load of the full-sized beam to be used in practice.

Let w = Weight or load carried by the experimental beam.

„ b = Breadth of the experimental beam in *inches*.

„ d = Depth „ „ „ „

„ l = Length „ „ „ „

„ W = Weight or load *to be* carried by the full-sized beam.

„ B = Breadth of full-sized beam in *inches*.

„ D = Depth „ „ „ „

„ L = Length „ „ „ „

Then, if the two beams are supported and loaded in exactly the same way, we have by proportion—

$$b : B :: w : W, \quad d^3 : D^3 :: w : W$$

$$L : l :: w : W$$

By combining these proportions we get--

$$bd^3L : BD^3l :: w : W$$

Or, $W \times bd^3L = w \times BD^3l$

$$W = w \frac{BD^3l}{bd^3L}$$

Note— wl/bd^3 therefore takes the place here of the constant k in the previous formulæ.

EXAMPLE I.—A bar of teak, 1 inch square and 12 inches between the supports, breaks with a load of 820 lbs. when hung at its centre. Find the breaking load at the centre of a bar of teak, 3 inches broad and 3 inches deep and 7 feet between the supports. If the bar be 2 inches broad instead of 3 inches, what should be its depth in order to support the same weight at the centre? (S. and A. Exam. 1888.)

ANSWER.—Here, *in the first case*, $b = 1''$; $d = 1''$; $l = 12''$; $w = 820$ lbs.; $B = 3''$; $D = 3''$; $L = 7' = 7 \times 12'' = 84''$.

Consequently, by the previous formula just deduced—

$$W = w \frac{BD^3l}{bd^3L}$$

$$W = 820 \times \frac{3 \times 3 \times 3 \times 12}{1 \times 1 \times 1 \times 84} = \frac{820 \times 3 \times 3 \times 3}{7} = 3162.8 \text{ lbs.}$$

In the *second case* we might quite easily substitute the numerical values as we did in the *first case*, and thereby arrive at the result; but it will be seen at once by the student, that since the only variable introduced into the second part of the question is the breadth, we have only to equate the breadth and depth to the constant load W . Thus, by calling $B_1 = 3''$; $B_2 = 2''$; $D_1 = 3''$; and D_2 the depth to be found, we have at once, from the above formula, since the load W and everything else are *constants*,

$$B_1 \times D_1^3 \text{ is proportional to } W;$$

$$B_2 \times D_2^3 \text{ is also proportional to } W;$$

$$\therefore B_1 \times D_1^3 = B_2 \times D_2^3$$

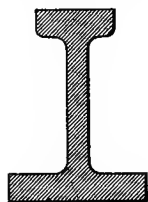
Or, $3'' \times 9 = 2'' \times D_2^3$

$$\therefore D_2^3 = \frac{3 \times 9}{2} = \frac{27}{2} = 13.5$$

$$\therefore D_2 = \sqrt[3]{13.5} = 3.674 \text{ inches.}$$

Different Sections of Cast-Iron, Wrought-Iron, and Steel Beams.—Having shown how the strength of a rectangular beam varies directly as the breadth B , and as the square of the depth D , it is natural, in the case of materials such as cast-iron, wrought-iron, and steel (which vary in regard to their resistance to extension and compression), that we should endeavour to show how these materials may be most economically disposed, so as to withstand the greatest load for a minimum of weight and cost. In the case of wooden beams, where it is found unprofitable to make them into any other shape, than the plain rectangular form—unless it be for purely architectural or artistic purposes—we only considered their strength when of that particular section; but it becomes quite another matter, when we have to consider the case of iron, for this material in its various modifications may be cast or rolled into any required shape, and therefore the weight and disposition of the material have a special bearing upon the cost and strength of iron beams.

Cast-Iron Beams or Girders.—The term girder is technically applied to beams of cast-iron, wrought-iron, or steel, which are used for spanning comparatively long distances, such as road or railway bridges, or large warehouses. As will have been observed from the table of ultimate strengths in Lect. I., Vol. II., the *ratio* of the ultimate strengths of cast-iron to compression and tension is as 45 to 7.5 tons; and further, since the stress on any material of which a beam may be composed is smaller and smaller the nearer the *neutral surface*, it is but natural that we should make the *upper flange* of a cast-iron beam *which is supported at both ends smaller* than the lower one, since the upper flange, being in compression, will obviously maintain a stress six times as great as an equal section in the lower side. On the other hand, if the cast-iron beam is of the *cantilever* type, where the upper side is subjected to tension and the lower side to compression, we should make the upper flange about six times as strong as the lower one. In practice, however, for the sake of obtaining a sound casting, (*i.e.*, having regard to the way in which the crystals of the metal naturally arrange themselves), the flange in tension is only made about four times the cross sectional area of the flange in compression as will be seen from the accompanying figure.



SECTION OF A
CAST-IRON BEAM.

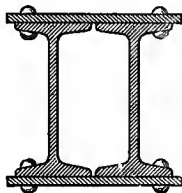
For vertical columns of cast iron, where the stress is purely one of compression, the **H** form is provided with flanges of the same dimensions at each end of the web, or, to come to the most

common form of strut—viz., that of the vertical cylindrical column—the section is that of a circle, or an **O**, since the stress is equally disposed throughout the cross section of the material.

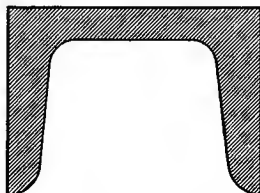
Wrought-Iron Beams and Girders.—Referring to the same Table of Ultimate Strengths and Working Loads in Lecture XXII., we see that the resistance to tension is 25 tons per square inch and to compression 20 tons, but from an average of a large



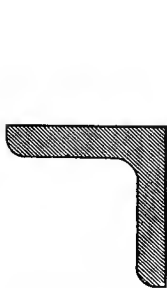
CROSS SECTION OF
A WROUGHT-IRON
BEAM.



CROSS SECTION OF
A WROUGHT-IRON
BOX GIRDER.



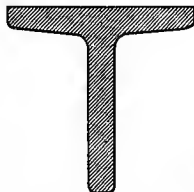
CROSS SECTION OF CHAN-
NEL IRON. — STRENGTH
IS LARGELY DUE TO THE
DEPTH OF THE SIDES.



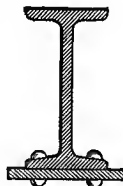
ORDINARY **L** IRON.



SPECIAL **L** IRON
FOR HEAVY
LOADS.



ORDINARY **T** IRON.



STEEL
BEAM.

CROSS SECTIONS OF WROUGHT-IRON AND STEEL BEAMS.

number of specimens it is found that the resistances to those two kinds of stresses is about the same. Consequently, the material may be disposed of equally between the top and bottom flanges, and remembering that the stress at the neutral surface is zero, we have only to connect the flanges with a sufficient thickness of metal to keep them together and to transmit the vertical shearing stresses. Wrought-iron beams and girders are therefore constructed of the forms shown by the first two figures above.

In special cases, such as ship-building and bridge-building, different forms of angle iron, T iron, and channel iron are used, as shown by the next four figures.

Steel Beams.*—Referring once more to the Table of Ultimate Strengths and Working Loads in Lect. I., Vol. II., we see that the resistance of steel to tension is only 45 tons per square inch of cross section, whereas it withstands 70 tons per square inch against compression. Consequently, in forming beams of this material, where the beam is supported at both ends, and the load is either placed in the middle or distributed, the bottom flange which is in tension must be made about double the cross area or weight of the top one, as shown by the last figure.

We have merely touched the fringe of this important subject on metal beams. We shall therefore have to return to it again in the Advanced Course.

* The Author is indebted to Messrs. P. and W. MacLellan, the well-known manufacturers of cast-iron, wrought-iron, and steel beams, for the full-size drawings of the various sections from which these reduced figures have been made by the Publishers, with the aid of photography.

LECTURE VII.—QUESTIONS.

1. A beam AD, 20 feet long, is supported at one end A, and at an intermediate point B, between its two ends, so that $AB=12$ feet and $BD=8$ feet. A weight of 5 tons rests at the end D, and a weight of 12 tons at a point C which is 8 feet from the hinged end A, determine the bending moments at C and B, and at a point midway between A and B. Draw the diagrams of bending moments and shearing force, and find the value of the shearing force at sections taken 2 feet from the free and hinged ends of the beam. *Ans.*

2. A beam of the same dimensions as in the previous question, is loaded over AB with a distributed load of 2 tons per foot-run. A weight of 10 tons rests on the free end D. Determine the bending moments at B, at centre of AB, and at a section 5 feet from the hinged end A. Draw the diagrams of bending moments and shearing force.

Ans.

3. A beam 10 ft. long is supported on two supports which are respectively 1 ft. and 7 ft. from the left-hand end of the beam. A weight of 10 lbs. hangs from the left-hand end of the beam, and a weight of 5 lbs. from the right-hand end. Find the two reactions and the bending moment and shearing force at the centre of the beam.

Ans. $R_1=9\frac{1}{2}$ lbs. ; $R_2=5\frac{5}{8}$ lbs. ; B.M. $=13\frac{1}{8}$ lbs.-ft ; S.F. $=\frac{5}{8}$ lb.

(C. & G., 1905, O., Sec. B.)

4. Determine the bending moments and shearing force at the centre of the beam, if the central portion is uniformly loaded in addition with 2 lbs. per foot-run. Draw the diagrams of bending moment and shearing force. *Ans.*

5. A beam ED, 40 feet long, is supported at two points A and B, so that $EA=8$ feet, $AB=20$ feet, $BD=12$ feet. Weights of 5 tons and 6 tons rest on the extremities E and D, and a weight of 10 tons at the centre of AB. Find the bending moments at A and B, and at the centre of AB.

Ans.

6. If in the previous example the central portion AB be loaded with a uniform load of 1 ton per foot-run, find the bending moments at the same places. *Ans.*

7. A beam 20 feet long, is arranged with symmetrical overhanging ends which are 4 feet from the supports, and loaded with a uniform load of 2 tons per foot-run over its whole length. Draw the diagrams of bending moments and shearing force. *Ans.*

8. Explain, by aid of a sketch, what are termed fixed or encasté beams ? Also, state what necessary precautions have to be observed when using these beams.

9. A beam, of uniform section, has its ends firmly fixed by being embedded in the abutments, and has a span of 20 feet. A load of 10 tons is resting at the centre. Find the bending moments at the ends and centre of the beam ; as well as at points distant 2 and 6 feet from one of the abutments. *Ans.*

10. A beam 30 feet long, of uniform section, is uniformly loaded with a weight of 4 cwts. per foot-run. If the beam be firmly fixed at the ends, determine the points of contrary flexure ; also, the bending moments at the ends, centre, and at a section taken 5 feet from the centre.

11. A wooden beam, supported at both ends and loaded in the middle, may have a saw-cut made upon the upper side without affecting the strength to an appreciable extent (if a wedge is inserted). How do you account for this result?

12. It will be observed that wooden beams are usually made rectangular in form, the depth being greater than the width. State the advantage that is derived from this form of arrangement, and the relation of strength in proportion to depth, width, and distance between the supports.

13. Uniform beams of timber of the same sizes are loaded and supported as follows: 1. Loaded at one end, and fixed at the other. 2. Fixed at one end, and uniformly loaded all over. 3. Supported at the ends, and loaded in the middle. 4. Supported at the ends, and loaded uniformly all over. 5. Fixed at the ends, and loaded in the middle. 6. Fixed at the ends, and loaded uniformly all over. What are their relative strengths? What are their relative stiffnesses? Where is each most likely to break?

14. Given that a rectangular rod of fir, 10 inches long, 1 inch broad, and 1 inch deep, and supported at both ends, will just sustain 540 lbs., when hung at its centre; what should be the depth of a bar of like wood 5 feet long and 2 inches broad, and supported at both ends, in order to support a load of $\frac{3}{2}$ of a ton when hung at its centre? *Ans.* 3.05 inches.

15. Two beams are of breadth 5 and 6 inches, and of depths 8 and 9 inches respectively; write down the numbers which represent their respective strengths in resisting transverse stresses.

Ans. 160:243.

16. Two wooden beams are each loaded in the centre and supported at the ends; one is solid and measures 8 inches $\times$ 8 inches in cross section; the other is made up of two beams each 8 inches broad and 4 inches deep, and placed one over the other so as to have the same sectional area as before. Will there be any difference in the breaking load of the beams, and if so, how much will it be? State the reasoning on which you rely.

17. A balk of timber, 11 ins. deep, is divided into three cuts of 2 ins., and 5 ins. thick. Compare the resistance to bending before and after cutting. *Ans.* 3:2:1. (C. & G., 1904, O., Sec. B.)

18. A beam of timber, rectangular in transverse section, is 2 inches broad, 3 inches deep, and 4 feet in length, and rests upon supports at its ends. The breaking load on the centre is 2000 lbs. What would have been the breaking weight if the beam had been 4 inches deep, 2 inches broad, and 4 feet between the supports, but loaded at a distance of 1 foot from the end? *Ans.* 4741 lbs.

19. Find the breaking weight at the centre of a beam of Memel fir, 12 inches deep, 10 inches wide, and 20 feet between the points of support. The breaking weight at the centre of a beam 1 foot long and 1 inch square is 545 lbs. *Ans.* 39,240 lbs.

20. A rectangular batten of fir, 6 feet in length, 2 inches broad, and 3 inches deep, is supported at its ends and can sustain a weight of 1100 lbs. when hung at the centre. If the load were distributed instead of hung at the centre, how much would the batten support? *Ans.* 2200 lbs.

21. A rectangular beam of timber supported at both ends, and of a given breadth and depth, just supports a load, W , at its centre. If the load be shifted to a point halfway between the centre and one end, by how much may the depth be reduced? *Ans.* 13.4 per cent.

22. Show, with sketches, the best forms and sections of flanged beams, (1) of cast iron, (2) of wrought iron, when supporting a load at the centre, and state the reasons which determine the particular forms.

23. Explain why iron girders are made with flanges at the top and bottom united by a web of metal, instead of being rectangular in section. What condition decides which of the two flanges shall contain the most metal?

24. Girders for carrying a load on their top flange, if of cast iron, have the section of metal on the bottom flange greater than on the top flange, but when made of wrought iron this rule does not hold. Why is this? Sketch a section of an ordinary cast-iron girder to carry a wall over a gateway, and of a wrought-iron plate girder for the same purpose.

25. What are the functions of the top and bottom booms and of the diagonal pieces of a railway girder? Why are the booms usually larger in section towards the middle of the girder, and the diagonal pieces usually larger towards the end of the girder? (S. E. B. 1901, 1902, and 1903.)

26. A beam of timber 2 feet long, 3 inches square, supported at the ends and loaded at the middle, breaks with a load of 7500 lbs. What load may be expected to break a beam of the same material, fixed at one end and loaded at the other, length 10', breadth 5", depth 9"? *Ans.* 5625 lbs.

27. In a timber beam of rectangular cross section, supported at the ends and loaded at the centre, how would the magnitude of the breaking load be affected in each of the following cases? (a) If the length were doubled; (b) If the breadth were doubled; (c) If the depth were doubled; (d) If the length, breadth, and depth were all doubled.

A beam 5 feet long, $2\frac{1}{2}$ inches broad, and 4 inches deep, supported at the ends, breaks with a load of 3500 lbs. at the centre. What uniformly distributed load would probably just break it if its length were 6 feet, breadth 3 inches, and depth 5 inches? *Ans.* 10,937 lbs. (B. of E. 1902.)

28. A beam, of rectangular section, is 9 inches deep, 3 inches broad, and 10 feet long. Find how much it will carry when loaded at the centre and supported at each end, the greatest intensity of stress allowed being 3 tons. What stress would be induced if the beam were laid flatwise and carried the same load? *Ans.* $W = 4.05$ tons; and $f = 9$ tons per square inch.

(C. & G. 1903, O., Sec. B.)

29. A beam of circular cross-section, 2 inches in diameter and 4 feet long between the supports, will stand without danger a load of 500 lbs. applied at the middle. How much would a beam 3 inches diameter of the same material and 6 feet long between supports take if also loaded at the centre? How much if uniformly loaded? *Ans.* $W = 1125$ lbs., and $wL = 2250$ lbs.

(B. of E., 1903.)

30. Wishing to know the proper breadth and depth of a beam of some new kind of timber to carry a certain load, you experiment with a small beam; describe how you would proceed and what use you would make of your results.

(B. of E., 1904.)

31. A timber beam, 6 inches wide, 10 inches deep, rests freely on two supports on the same horizontal level, which are 20 feet apart. Neglecting the weight of the beam itself, find what load placed in the centre would be required to break it. A test bar 1" by 1" in section on a span of 20" is broken by a load in the centre of 350 lbs. *Ans.* $W = 7.9$ tons.

(B. of E., 1905.)

32. Compare the strengths of two beams, each loaded at the centre, and each of rectangular section, 6" by 3"; the first one having the larger side vertical and of 20' span, the second having the shorter side vertical and of 10' span. *Ans.* Equal.

(C. & G., 1905, O., Sec. B.)

N.B.—See *Appendices B and C* for other questions and answers.

LECTURE VIII.

CONTENTS.—Stresses in Structures—Culman's Method of Sections—Application to a Loaded Beam—Example of the Application of Ritter's "Method of Sections" to a Roof Truss—Graphical Determination of the Maxima and Minima Stresses in Curved Trussed Girders due to Live Loads—Loading for Railway Bridges—Relation of Curvature, Slope, and Deflection of Beams—Example I.—Stiffness of a Beam or Girder—Steel Girders—Plate Girders—Flanges—Parabolic Girders—The Web of Girders—Function of the Web in Resisting the Shearing Force with Parallel Girders—Function of the Web in Resisting the Shearing Force with Parabolic Girders—Suspension Bridges—Definition of a Suspension Bridge—Method of Attaching the Main Chains to the Piers, and Finding the Pressure on the Piers in Suspension Bridges—Example II.—Stiffening Suspension Bridges by means of Girders—Single Girder Continuous from Pier to Pier under a Uniform Live Load—Suspension Bridges with the Stiffening Girder Hinged at the Centre—Questions.

Stresses in Structures.—The determination of the stresses in structures such as trusses or girders is based upon the following principles:—

1st. The principle of the resolution of forces.

2nd. The principle of moments.

The former may be stated thus:—If any number of forces in the same plane act at a point, or at different points, of a rigid body and are in equilibrium, the algebraic sum of all their components in any direction is zero—*i.e.*, the tendency to move the body in one direction is exactly balanced by an equal tendency in the opposite direction.

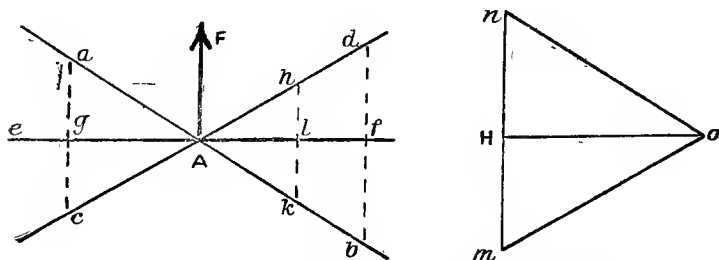
The second principle may be stated as follows:—If any number of forces in the same plane act at a point, or at different points, of a rigid body and are in equilibrium, the algebraic sum of the moments of these forces, taken with reference to any point in their plane, is zero.

Hence, the tendency to produce rotation of the body in one direction is exactly balanced by an equal tendency to produce rotation in the opposite direction.

These two fundamental principles give rise to two methods of calculation, and each may be applied analytically or graphically, but it will generally happen in any particular problem, that one method offers advantages in simplicity over the other.

Culman's Method of Sections.—Suppose we have a single force F acting at a point A . The force polygon for this particular case is represented in magnitude and direction by the straight line mn . Select any point O as a pole, and join it with the points n and m . This is equivalent to resolving the force F in two directions. Through the point A , draw two lines ab and cd , parallel to the lines nO and mO respectively. Draw a line OH perpendicular to the line mn . Then the distance OH is called the *polar distance*. Also draw through A a line ef parallel to OH , and the dotted lines ac , hk , db parallel to the direction of the force F .

Then the moment of the force F , with reference to any



EXPLANATORY DIAGRAM OF CULMAN'S METHOD OF SECTIONS.

point g , is $AF \times Ag$. But in these figures we have, by similar triangles—

$$AF : OH :: ac : Ag.$$

$$\therefore AF \times Ag = OH \times ac.$$

That is, the moment of the force AF , with reference to any point, is equal to the ordinate drawn through this point parallel to AF , included by the two components into which the force AF is resolved, multiplied by the polar distance OH in the force polygon.

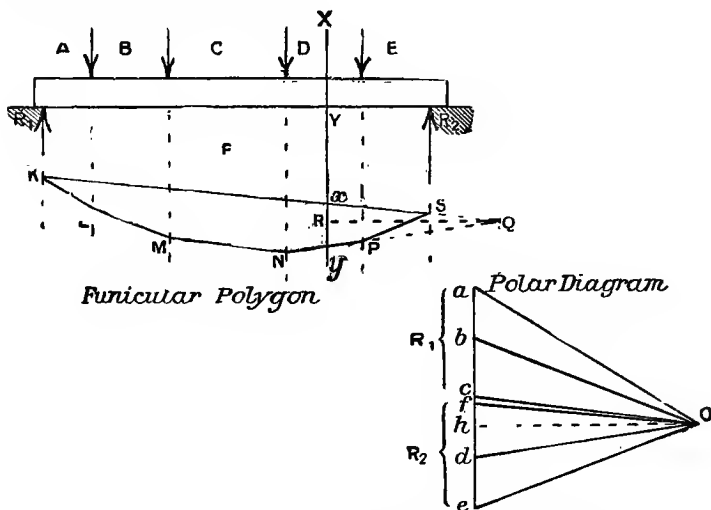
The same principle may be applied to represent the moments of any number of forces.

Application to a Loaded Beam.—Let us take the case of a beam resting at both ends and supporting the vertical loads AB, BC, CD, and DE.

First, letter the frame diagram according to Bow's system (as used in this book).

Second, draw the polygon of forces or the line of loads ae , and take any pole O at a convenient distance from the line ae . Join the pole O with the points a, b, c, d , and e on the line of loads.

Third, then draw the funicular polygon $KLMNP$ S. The line KS is the closing line of the funicular polygon. The reactions R_1, R_2 —i.e., AF and FE at the supports—can be determined by drawing the line OQ in the polar diagram parallel to the closing line KS of the funicular polygon. The lengths of af and fe , measured to the scale of loads, give the magnitudes of R_1 and R_2 respectively.



APPLICATION OF CULMAN'S METHOD TO A LOADED BEAM.

To determine the Bending Moment at any Section, say at the line XY of the beam.—As stated in Lecture XXVIII., the bending moment at any section is the moment of the resultant of all the forces which act on one side or other of the section; then, take the forces acting on the left-hand side of the section line XY. These are FA, AB, BC , and CD , and their resultant is $FA - AB - BC - CD = fa - ab - bc - cd = -fd$. Consequently, a point in the line of action of this resultant is given by the intersection of those sides of the funicular polygon which occupy the spaces F and D ; that is, of the sides cut by the vertical line $XYxy$, drawn through the section at XY . Let these sides KS and NP when produced, intersect in the point Q . Draw through the point Q a line QR , perpendicular to $XYxy$. Then, the magnitude of the bending moment at the section XY is $fd \times QR$. But in the triangles Qxy, Ofd , the sides Qy, Qx, xy are

respectively parallel to the sides Od , Of , and fd . Hence, the triangles Qxy and Ofd are similar. But QR and Oh are the perpendiculars on corresponding sides from the opposite angles of the triangles;

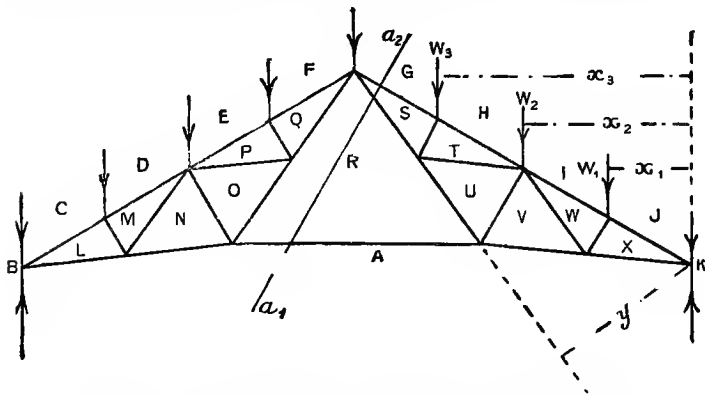
$$\therefore xy : QR :: fd : Oh;$$

$$\therefore QR \times fd = XY \times Oh.$$

Thus, we see, that the *bending moment at any section of the beam* is got by *multiplying the ordinate of the funicular polygon at that point by the polar distance.*

In consequence of this important point, the funicular polygon for a loaded beam is often called the **Bending Moment Diagram**.

Professor Ritter's Method of Moments.—The principle of Professor Ritter's "method of moments" or "method of sections," as it is sometimes called, may, with advantage, be applied to determine the stresses in structures, such as roof trusses and girders. The principle of the equality of moments may be extended to any number of forces, but the sum of the moments of the forces tending to cause motion in one direction, must always be equal to the sum of the moments of the forces tending to cause motion in the opposite direction. By an amplification of this ordinary method of moments we can determine not only the stresses in the flanges of a bowstring girder, but also the stresses in the lattice bars. This method is equally applicable for obtaining the several stresses in the members of roof trusses.



SHOWING THE APPLICATION OF PROF. RITTER'S METHOD OF MOMENTS OR SECTIONS.

The process of working this method is as follows:—

(1) Draw the frame diagram, and indicate the direction of all external forces by arrow-heads, deducting the downward loads at end joints from the upward reactions and substituting the nett upward forces.

(2) Draw "lines of section," each to cut the truss in not more than three of its members. These lines of section divide the truss into two parts, one of which is supposed to be removed, and the external forces acting upon the remaining portion of the truss are only to be considered.

A sufficient number of sections must be taken to include all the members of the truss.

(3) The stress upon any one of the three members, cut by the line of any section, may be calculated by taking moments round the point of intersection of the other two. It will be seen, that by adopting this method, the moments of the stresses in the members which meet at the point above indicated, become zero. Consequently, it only becomes necessary to equate the moment of the stress in the first member to the algebraical sum of the moments of the external forces acting on the portion of the truss considered. The sign (+) may be used for the moments of forces tending to cause rotation from left to right—i.e., in the direction of the hands of a watch, and (−) for those moments turning in the opposite direction.

Let it be required to find by the “method of sections,” the magnitude of the force acting along the bar RS in the roof truss shown.

Draw a line of section $a_1 a_2$ through the truss to cut three members. Then, in order to keep the structure in equilibrium three forces A R, R S, and S G, must be applied at the ends of the members cut by the line of section. Let the weights W_1 , W_2 , and W_3 at the joints on the right-hand side of the truss, and at distances x_1 , x_2 , and x_3 from the vertical passing through K, the heel of principal. Then, taking moments about the point K, it will be seen, that the forces acting along the bars A R and S G pass through the point K; and, therefore, have no moment about it, but that the force R S has a moment about the point K equal to R S $\times$ y .

$$\text{Hence,} \quad RS \times y = W_1 x_1 + W_2 x_2 + W_3 x_3.$$

$$\text{Or,} \quad \text{The stress in RS} = \frac{W_1 x_1 + W_2 x_2 + W_3 x_3}{y}.$$

Example of the Application of Ritter's “Method of Sections” to a Roof Truss.—Roof 30 feet span carried on king trusses 10 feet apart, the rafters and struts of which are inclined at 30° with the horizontal. Taking the vertical load on the roof at 60 lbs. per square foot of horizontal surface covered, determine the stresses in each bar of the truss.

The total vertical load = span $\times$ distance between trusses $\times$ wt. per sq. ft.

$$,, \quad ,, \quad = 30 \times 10 \times 60 = 18,000 \text{ lbs., or } 8 \text{ tons.}$$

The joints A and F each carry 1 ton. Each of the joints B, C, and D of the truss carry 2 tons. Omitting the loads carried by the supports, the effective reactions R_1 and R_2 are each equal to 3 tons.

Let the stresses in the bars BH, HG, CI, IH, and IK of the frame, be denoted by the letter S with the respective letters of the bar as a suffix—i.e., S_{BH} , S_{HG} , &c.

1. To find the Magnitude of the Stresses S_{BH} and S_{HG} .—By resolving the effective reaction horizontally and vertically at a vertical section cutting the bars BH and HG, we have—

$$R_1 = S_{BH} \sin 30^\circ, \text{ and } S_{HG} = S_{BH} \cos 30^\circ.$$

Therefore—

$$S_{BH} = \frac{R_1}{\sin 30^\circ} = \frac{3}{.5} = 6 \text{ tons, compression.}$$

$$S_{HG} = S_{BH} \cos 30^\circ = 6 \times .866 = 5.196 \text{ tons, tension.}$$

2. To find the Magnitude of the Stresses S_{CI} and S_{IH} .—Draw a section line cutting the bars CI , IH , HG ; then the stresses S_{CI} , S_{IH} , and S_{HG} in the members must balance the external forces of 3 tons and 2 tons in order that equilibrium may be preserved. Taking moments about the point G where the bars IH and HG meet—i.e., at the foot of the king post, we find—

$$S_{CI} \times BG \cos 30^\circ = R_1 \times HG - BC \times \frac{1}{2} HG.$$

$$,, \quad ,, \quad = 3 \times 15 - 2 \times 7.5 = 30 \text{ ton-feet.}$$

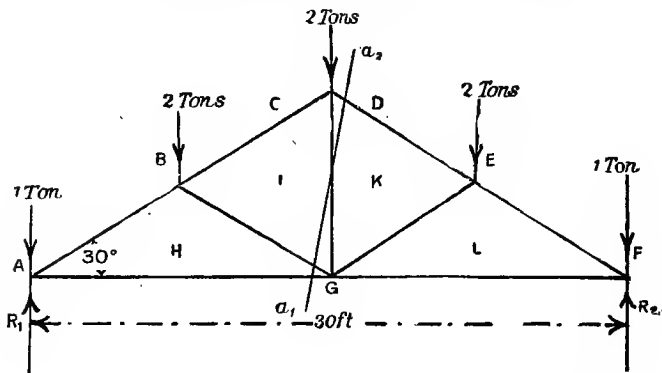
$$\text{But,} \quad BG = 15 \tan 30^\circ = 15 \times .5774 = 8.661 \text{ feet.}$$

$$\therefore \quad S_{CI} = \frac{30}{8.661 \times .866} = 4 \text{ tons, compression.}$$

Resolving horizontally, we get—

$$S_{CI} \cos 30^\circ + S_{HI} \cos 30^\circ - S_{HG} = 0.$$

$$\therefore \quad S_{HI} = \frac{S_{HG} - S_{CI} \cos 30^\circ}{\cos 30^\circ} = \frac{5.196 - 4 \times .866}{.866} = 2 \text{ tons, compression.}$$



BITTER'S METHOD OF SECTIONS APPLIED TO A KING TRUSS.

3. To find the Magnitude of the Stress S_{IK} .—Take an approximately vertical section $a_1 a_2$ passing to the right of CD and to the left of G .

From the symmetry of the truss and the loading of same, the stress in bar DK is equal to the stress in the bar CI .

Resolving vertically—

$$S_{CI} \sin 30^\circ + S_{HI} \sin 30^\circ - S_{IK} + 3 - 2 - 2 = 0.$$

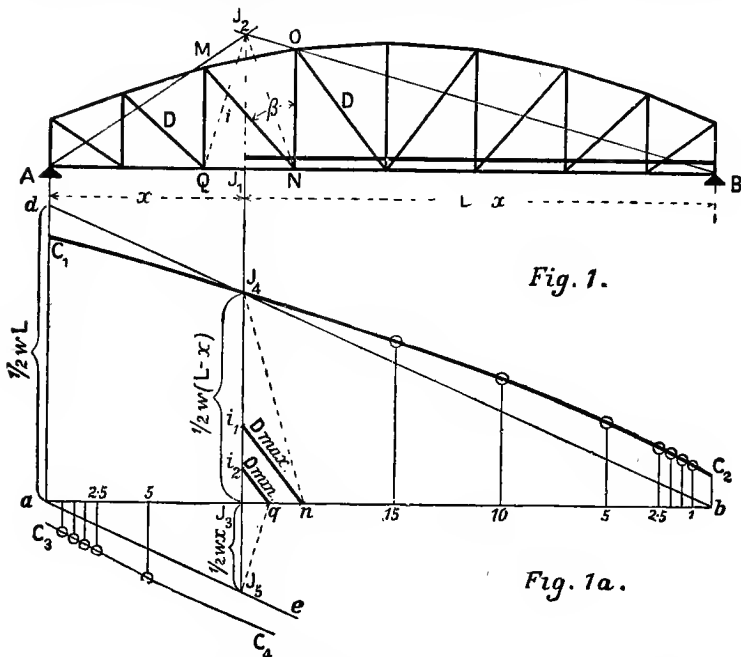
$$S_{IK} = (S_{CI} + S_{HI}) \sin 30^\circ - 1.$$

$$S_{IK} = (4 + 2) \times .5 - 1 = 2 \text{ tons, tension.}$$

The truss being loaded symmetrically, the stress in the corresponding bars of each half of the truss will be exactly the same.

Graphical Determination of the Maxima and Minima Stresses in Curved Trussed Girders due to Live Loads.*—Trussed girders with curved booms, especially those which have one curved and one straight boom, are frequently used in bridge building for both large and small spans.

The following graphic method of finding the magnitude of the stresses may prove of interest, owing to its simplicity, accuracy, and general application, since the stresses can, by means of a few lines, be found in any member for any position of the live or rolling load.



TRUSSED GIRDER WITH CURVED TOP AND STRAIGHT BOTTOM BOOMS.

1st. *To Find the Stresses in Diagonals.*—Let w represent the uniform live or rolling load per unit length of span; then, to find the stress in the diagonal MN, Fig. 1, first find the central load point J_2 at the intersection of AM and BO produced, and from this point draw the load partition $J_2 J_1$ perpendicular to the bottom boom. The load partition is that position in which the rolling load must be placed to produce no transverse stress on that bay where we intend to find the stresses in the diagonals or verticals.

* I am indebted to Mr. J. M. Ringquist, of Stockton-on-Tees, for kindly supplying drawings and manuscript for this new article on "Curved Bridge Trusses" for this book.—A. J.

The load partition $J_1 J_2$ divides the diagonal $M N$ into two parts, $M i$ and $N i$, so that they represent the maximum and minimum stress respectively in this diagonal due to the rolling load, when the length $J_1 J_2$ is made to represent half the total load—i.e., $J_1 J_2 = \frac{1}{2} w x$ or $\frac{1}{2} w (L - x)$.

In order to be able to scale off these maximum and minimum stresses, we need only construct similar triangles $n J_2 J_1$ and $q J_2 J_1$ to the triangles $N J_1 J_2$ and $Q J_1 J_2$ in the frame diagram. In these triangles, the verticals represent the unit lines of loads on $B J_1$ or $A J_1$ respectively—i.e., equal to $\frac{1}{2} w (L - x)$ or $\frac{1}{2} w x$, when the line $J_4 n$ is drawn parallel to $J_2 N$ and $J_5 q$ parallel to $J_2 Q$. Hence, we are now able to scale off from the lines $n i$ and $q i$ the maximum tension and compression stresses in the diagonal $M N$, if these are drawn parallel to the diagonal $M N$.

There are different ways of proving the foregoing, but the following method will, perhaps, be most suitable:—

Supposing that the boom AB was only loaded from the end B to the point N by the load $w \times BN$, for which we can introduce two loads each equal to $\frac{1}{2} w \times BN$ acting at the points N and B of the beam.

If we now draw a line through the panel $MONQ$ which will intersect the top flange or boom between M and O , the diagonal $M N$ or D , and the bottom boom between Q and N , and imagine that the girder is cut into two parts (as in *Ritter's "Method of Sections,"* previously described); then, the forces acting on the right- and left-hand sides of this intersection line must be in equilibrium. Consequently, the resultant $A M J_2$ of the horizontal stress in AB , and the reaction on the left abutment at A , must go through the point of intersection of the lines $N M$ and $O M$ at the point M . Also, the resultant $B O J_2$, due to the stress in BA and the horizontal component of the stress in the diagonal $M N$ or D , and reaction at the right-hand abutment $B = \frac{1}{2} w \times BN$, must, likewise, pass through the point O or intersection of the top boom, and the vertical forces $\frac{1}{2} w \times BN$ acting at the point N , as well as the vertical component $D \cos \beta$ of the stress in the diagonal $M N$ or D . In consequence of these facts, we have one resultant acting in the direction $A M$, and the other resultant acting in the direction $B O$. These two resultants can now be united into one force which will act in the direction $J_2 N$. This latter force $J_2 N$ is again the resultant force from $\frac{1}{2} w \times BN$ and the horizontal component of the force acting along the diagonal D . Therefore, if we make the length of the line $J_1 J_2$ to scale, equal to $\frac{1}{2} w \times BN$, then the length of the line $N i$ to same scale, would represent the magnitude of the stress in the diagonal D .

However, to find the maximum stress in the diagonal D , the part of the truss, or girder, or span $N J_1$ must still be considered loaded. If we resolve this force $w \times N J_1$ into two forces, each equal to $w \times N J_1 \div 2$ acting at the apices Q and N , we again find, that the line $A M$ represents the direction of the resultant from the forces O , D , and $w \times N J_1 \div 2$. Likewise, the line $B O$ represents the direction of the resultant from the forces O , $D \cos \beta$, and $w \times N J_1 \div 2$. The resultant of these last two resultant forces is again co-equal to the resultant from the forces $D \sin \beta$ and $w \times N J_1$. And, as the direction of this force is given by the two points—viz., by the point J_1 , and by the point at which the force $w \times N J_1$ acts, it is determined by the length $(N J_1 \div 2)$ from the joint N .

Hence, if we make the length of the line $J_1 J_2$ to represent $w \times N J_1$, then $N i$ will represent double the stress produced in the diagonal D by the load on the length $N J_1$. Therefore, we find the stress in the diagonal for a load extending from B to $J_1 = w (B N + N J_1)$ is represented to scale by the length of the line $N i$, if $J_1 J_2 = \frac{1}{2} w B N + \frac{1}{2} w \times N J_1 = \frac{1}{2} w \times B J_1$.

When the length $A J_1$ of the beam is loaded, then a similar proof to that just shown for the maximum stress can be given for the minimum stress produced.

2nd. To Find the Stress in the Vertical Bar $Q M$, Fig. 2.—Having first determined the central load point J_1 , now draw through it a line parallel to the diagonal $N M$ —i.e., at an angle β with the vertical.

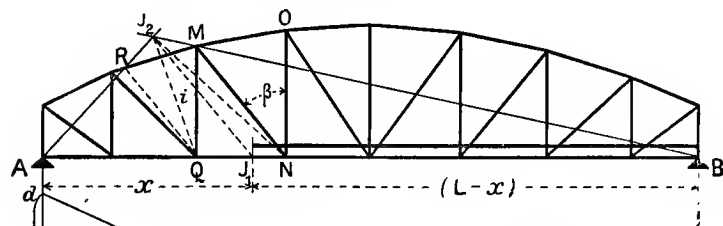


Fig. 2.

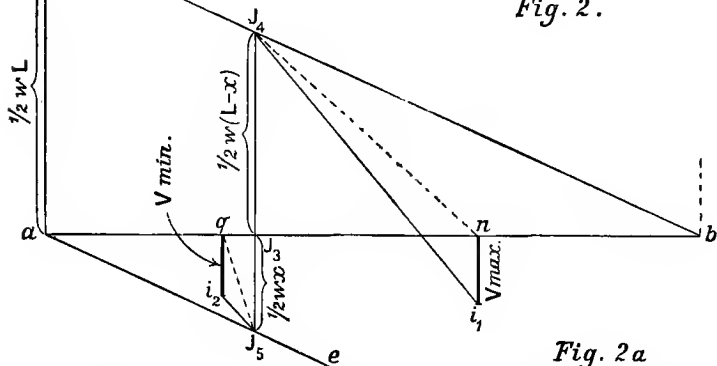


Fig. 2a

TRUSSED GIRDER WITH CURVED TOP AND STRAIGHT BOTTOM BOOMS.

The vertical bar $Q M$ is divided into two parts $Q i$ and $M i$, the first gives the minimum and the latter the maximum stress produced in that bar if the length of the line $J_1 J_2$ is made by scale to represent half the load multiplied by $\sec \beta$ —i.e., equal to $\frac{1}{2} w \times A J_1 \times \sec \beta$, or $\frac{1}{2} w \times B J_1 \times \sec \beta$ respectively. To be able to scale off these stresses from a diagram, we must draw $J_4 n$ parallel to $J_2 N$ and $J_5 q$ parallel to $J_2 Q$, at the same time making the length of line $J_3 J_4 = \frac{1}{2} w \times B J_1$, and $J_3 J_5 = \frac{1}{2} w \times A J_1$. Further, through the points J_4 and J_5 , draw the lines $J_4 i_1$ and $J_5 i_2$ parallel to the diagonal $M N$ in the frame diagram; then, the vertical $n i_1$ will represent the maximum stress, and the vertical $q i_2$ will represent the minimum stress in the vertical bar.

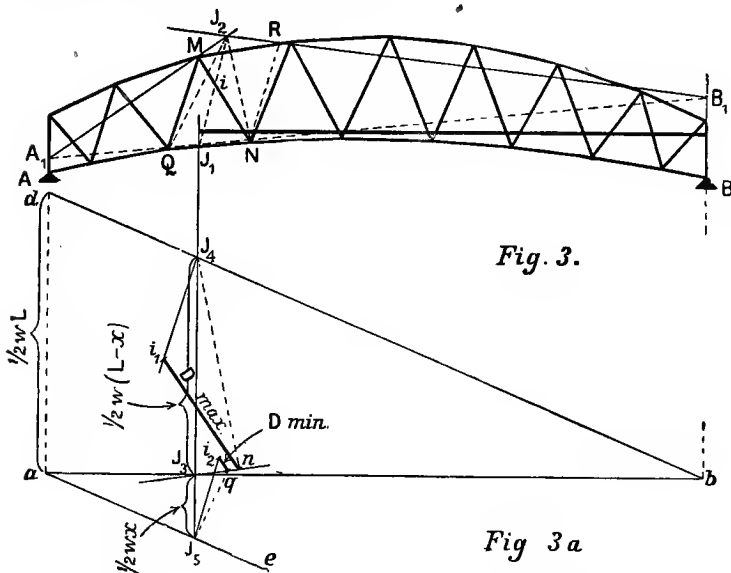
The correctness of this diagram can be proved in a similar way to that given under the first part. We imagine the boom to be loaded up to the point N only, and resolve this load into two forces $\frac{1}{2} w \times B N$ acting at the points N and B . We then intersect the panel by passing an intersection

line through the vertical bar V , the top boom O , and the bottom boom U . The forces acting on each side of the intersection line can be either resolved horizontally into the bottom boom, or in the direction of the diagonal NM .

Then, the resultant of the force in O and $V \sec \beta$ must act in the direction AR ; also, the resultant from O , V , and $\frac{1}{2}wBN \times \sec \beta$ will act in the direction BM . Whilst the resultant from both these two forces again must go through the points J , and N .

This resultant is equal to the resultant from the component of V , acting in the direction of the bottom boom, and the force $\frac{1}{2}wBN \times \sec \beta$, acting in the direction MN . Consequently, if the length of the line J_1J_2 be made to represent $\frac{1}{2}w \times BN \sec \beta$, then J_1N will represent the value of $V \tan \beta$, and $Mi = V$, the stress in the vertical bar.

Now, we have to add the load on the part NJ_1 of the truss or girder, and again resolve the load $w \times NJ_1$ into two loads $\frac{1}{2}w \times NJ_1$, each acting at the points Q and N . We shall find, reasoning as above, that the length of the line Mi denotes the magnitude of the stress in the vertical bar if we make $J_1J_2 = \frac{1}{2}w \times NJ_1 \times \sec \beta$. For the whole load on $BJ_1 = w(L-x)$, and we must introduce the length of line $J_1J_2 = \frac{1}{2}w(L-x) \sec \beta$ to find in Mi the maximum stress in the vertical bar. The minimum stress Qi is found in a similar way.



TRUSSED GIRDER WITH CURVED TOP AND BOTTOM BOOMS.

3rd. *Generalisation of this Method to other Forms of Trusses, for example, Fig. 3.*—The above method of determining the maximum stresses on the bar of braced girders is also applicable to trussed beams in general. We

find the maximum stress in an analogous way to the preceding case, by drawing a line through the central load point of the loaded boom J_1 , parallel to that bar MQ , which connects the bar MN with the following apex of the loaded boom. The bar MN is divided into two parts Ni and Mi , which represent the maximum and minimum stresses on that bar, if the length J_1J_2 corresponds to the value of the components $\frac{1}{2}w(L-x)$ or $\frac{1}{2}wx$ respectively. The minimum stress can be found by resolving the load into the direction of the bottom boom and into that of the bar QM . Consequently, we have the diagram, as shown in Fig. 3a, in which $J_3J_4 = \frac{1}{2}w(L-x)$ and $J_3J_5 = \frac{1}{2}wx$. Draw through J_3 a line parallel to the bottom boom bar QN , also J_4n parallel to J_3N , and J_5q parallel to J_2Q . Then, draw J_4i_1 and J_5i_2 as parallels through J_4 and J_5 to the bar MQ . Likewise, draw ni and qi through the points n and q parallel to the bar MN , to ultimately find the lengths ni_1 and qi_2 , which represent to scale the desired maximum and minimum stresses.

4th. The rolling or live load w has to be chosen in accordance with the loaded length of the girder. There is no difficulty in always introducing into the above construction that rolling or live load which is the substitute in a single load diagram for a fixed loaded length $(L-x)$.

Loading for Railway Bridges.—It is difficult to obtain any summarised information on the subject of the loading of bridges, as the various railway companies, both in Great Britain and on the Continent, have different loadings, and their locomotives seem to be in a state of transition, for they are now made much heavier than they were only a short time ago.

In order to give the student a little knowledge of what is required, the following table is appended, giving the equivalent uniform live load for an American engine with tender, weighing 136 tons at 2,000 lbs. per ton, covering a length of 104 feet, followed by a train weighing 4,000 lbs. per foot, taken from a diagram in Waddell's "De Pontibus":—

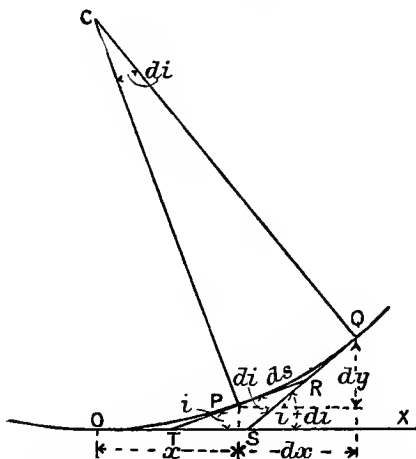
Span in Feet,	100	120	140	160	180	200	220	240	260	300	400
Equivalent Uniform Load in Lbs.,	5,300	5,220	5,150	5,080	5,010	4,940	4,880	4,830	4,775	4,600	4,550

The loads given in the table are plotted as ordinates to the corresponding abscissæ, and the curve c_1c_2 , shown in the stress diagram of Fig. 1, is obtained. This curve is substituted for the straight line ab .

Relation of Curvature,* Slope, and Deflection of Beams.†—

If i denote the circular measure of the inclination or slope of the beam at a distance x from the origin of co-ordinates.

Then,
$$i = \tan i = \frac{dy}{dx}.$$



SKETCH ILLUSTRATING THE RELATION OF CURVATURE,
SLOPE, AND DEFLECTION OF BEAMS.

Because the deflection is very small, dx may be taken as sensibly equal to ds .

But, the curvature
$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M}{EI}.$$

Therefore, the slope
$$i = \frac{dy}{dx} = \int \frac{M}{EI} \cdot dx,$$

And,
$$y = \int \int \frac{M}{EI} \cdot dx^2.$$

* **Curvature.**—The curvature of a circle is the reciprocal of its radius, or it is the angular change of the direction of the curve per unit of length; and of any curve it is the curvature of the circle which most nearly agrees with the curve.

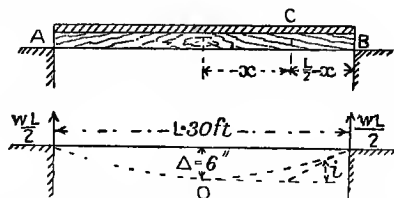
† See *Proc. Inst. C.E.*, vol. cxlix., 1902, for paper on "The Theory of Cast-iron Beams," by Edward V. Clark, B.Sc., Stud.Inst.C.E.

E is a constant for the same material, and I is a constant if the section of the beam is uniform. Also M can be expressed in terms of x , so that the above equations may be written—

$$i = \frac{1}{EI} \int M \cdot dx, \text{ and } y = \frac{1}{EI} \int \int M \cdot dx^2.$$

EXAMPLE VII.—A beam of uniform section is supported at the two ends, 30 feet apart, and is found to bend 6 inches downwards in the middle under its own weight. Determine the slope of the beam at the points of support. Find also what the slope would be if the same deflection of 6 inches were produced by the imposition of a central load instead of a uniform load. (I.C.E., Oct., 1898.)

ANSWER.—(a) Let AB be the beam of length L , resting on supports at A and B , and loaded with w lbs. per foot in length. Take the origin at the middle of the beam in order to make the constants of integration zero, since $i = 0$ when $x = 0$.



BEAM OF UNIFORM SECTION AND LOADING.

The bending moment M at a distance x from O is—

$$M = \frac{wL}{2} \left(\frac{L}{2} - x \right) - w \left(\frac{L}{2} - x \right) \left(\frac{\frac{L}{2} - x}{2} \right) = \frac{w}{2} \left(\frac{L^2}{4} - x^2 \right).$$

$$\therefore \frac{1}{\rho} = \frac{d^2 y}{dx^2} = \frac{w}{2EI} \left(\frac{L^2}{4} - x^2 \right).$$

Integrating, we get the slope at any point of the beam—

$$i = \frac{dy}{dx} = \frac{w}{2EI} \left(\frac{L^2 x}{4} - \frac{x^3}{3} \right).$$

The greatest slope at ends is found by substituting $x = \frac{L}{2}$, we have—

$$i = \frac{wL^3}{24EI}.$$

Also, to find the deflection—

$$y = \int i dx = \frac{w}{2EI} \int_0^x \left(\frac{L^2 x}{4} - \frac{x^3}{3} \right) dx = \frac{w}{2EI} \left(\frac{L^2 x^2}{8} - \frac{x^4}{12} \right).$$

y is greatest at the ends—i.e., when $x = \frac{L}{2}$.

Therefore, substituting this value, we obtain the deflection at the ends—

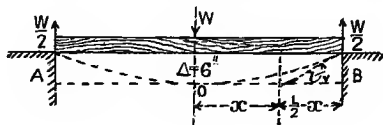
$$y = \frac{1}{2} = \frac{w}{2EI} \left(\frac{L^2 x^2}{8} - \frac{x^4}{12} \right) = \frac{5wL^4}{384EI} = \frac{5}{384} \frac{WL^3}{EI}.$$

This represents the rise at the ends, which is equal to the sag at the middle.

Hence, in this example, when the beam is uniformly loaded—

$$i = \frac{w L^3}{24 EI} = \frac{192}{24 \times 5 L} = \frac{16}{10 L} = \frac{16}{10 \times 30} = \frac{4}{75} = \cdot 053 = \tan 3^\circ.$$

(b) When the beam is supported at both ends, and loaded with a single load W at the centre of the span which produces the same deflection of 6 inches.



BEAM OF UNIFORM SECTION WITH A CENTRAL LOAD.

Taking the origin at the middle of the beam, in order to make the constants of integration zero. Then, since $i = 0$ when $x = 0$, we have—

$$\text{Bending moment } M = \frac{W}{2} \left(\frac{L}{2} - x \right).$$

$$\text{And, therefore,} \quad \frac{1}{\rho} = \frac{d^2 y}{dx^2} = \frac{W}{2 EI} \left(\frac{L}{2} - x \right).$$

Integrating, we get the slope at any point—

$$i = \frac{dy}{dx} = \frac{W}{2 EI} \left(\frac{L}{2} x - \frac{x^2}{2} \right).$$

At the ends, where $x = \frac{L}{2}$, the slope—

$$i = \frac{W L^2}{16 EI}.$$

$$\text{To find the deflection,} \quad y = \int i dx = \frac{W}{4 EI} \int (Lx - x^2) dx = \frac{W x^2}{4 EI} \left(\frac{L}{2} - \frac{x}{3} \right),$$

and its greatest value—namely, the rise of the ends of the beam above the middle—will be got by substituting $x = \frac{L}{2}$.

$$\therefore y = \frac{W L^3}{48 EI} = \frac{1}{2}.$$

Consequently, from these last two equations, we have—

$$i = \frac{W L^2}{16 EI} = \frac{24}{16 L} = \frac{3}{2 L} = \frac{3}{2 \times 30} = \frac{1}{20}.$$

$$\therefore i = \tan 2^\circ 50'.$$

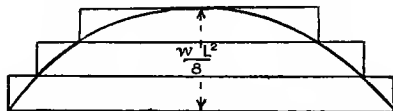
Stiffness of a Beam or Girder.—The stiffness of any beam or girder is measured by the ratio of its *maximum deflection to its span*. In practice, this ratio varies from about $\frac{1}{1200}$ to $\frac{1}{2500}$ in long steel spans for bridge work. Whilst for short girders and rolled joists it is about $\frac{1}{800}$ to $\frac{1}{1000}$.

Steel Girders.—In steel girders the two flanges are held apart by a thin, deep web, as in the plate girder type, or by diagonal bracing as in lattice girders. In actual practice, when designing a girder, the flanges alone are taken as resisting the bending moment, and the cross-sectional area of the web is neglected in finding the moment of resistance of the cross-section. The bending stresses are borne mainly by the flanges, the one is in tension and the other in compression, and as the thickness of each flange is small in comparison to the total depth of the girder, the intensity of stress is practically uniform throughout the whole of each flange.

The function of the web or diagonal braces is to resist the whole of the shear stresses.

The determination of the bending moment diagrams for the several methods of loading a girder are given in Lectures II. and III., Vol. II. Consequently, the bending moment at any point of the girder, divided by the corresponding depth of the girder, gives the total stress in the flanges at that point, and this result, divided by the maximum intensity of the stress, gives the net area of one flange.

Plate Girders.—Flanges.—In the rectangular plate girder the total flange stress is greatest at the middle, and diminishes towards the abutments, as seen by plotting a bending moment diagram. Consequently, the section of the flanges should correspondingly diminish from the centre to the ends of the girder. This is accomplished in actual practice by keeping the width of the flanges the same throughout the length of the girder, and reducing the thickness by reducing the number of plates. The bending moment diagram then lends itself very readily to the correct stepping of the plates.



BENDING MOMENT DIAGRAM USED TO DETERMINE THE LENGTHS OF FLANGE PLATES FOR A GIRDER.

Suppose it was found, that three thicknesses of plate were required to resist the bending stresses in the flanges at the middle of the girder; then, if the depth of the bending moment diagram be divided into three strips of equal thickness, each strip will represent one plate. Hence, if the intersection of these strips with the bending moment curve are projected up on to the flange of the girder, it at once gives the position where the plates may be stepped. Although it is usual to allow the plates to overlap from 6 inches to 12 inches beyond the points thus obtained by the bending moment diagram.

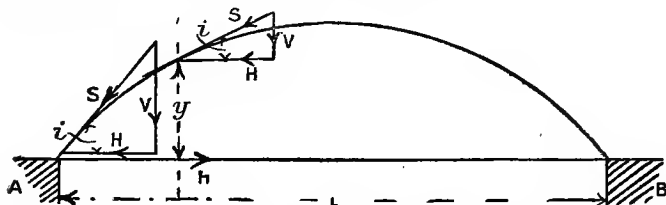
Parabolic Girders.—If the horizontal stress in either flange be constant, then the depth of girder d varies as the bending moment, and therefore the shape of the curved flange of the girder will be that of the bending moment diagram, which is a parabola.

The horizontal component of the inclined stress at any point in the curved flange, or the uniform horizontal stress in the lower flange may be obtained by calculation from the formula $M = \frac{w L^2}{8 d} = H$, the horizontal stress in the flange. The vertical component of any point in the curved flange, $V = H \tan i$.

The total stress at any point in the curved flange is—

$$\frac{S}{H} = \frac{1}{\sin i} = \sec i. \quad \therefore S = H \sec i.$$

Hence, the total stress S will vary as $\sec i$, and, consequently, a greater thickness is required at the ends than at the centre, because $\sec i$ increases as i increases.



SHOWING THE INCREASE IN FLANGE STRESS IN A PARABOLIC GIRDER.* (*)

The Web of Girders.—The shearing force in the web can be readily obtained from the shearing force diagram.

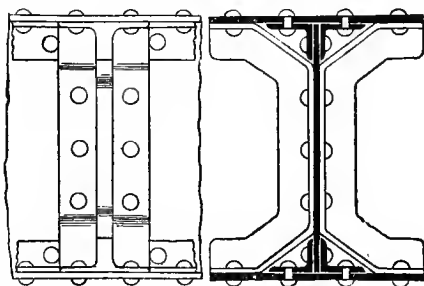
Let SF = Shearing force at any point in tons.

„ f_s = Working or permissible shear stress in tons per square inch
(it does not usually exceed 2 tons per square inch).

„ d_w = Depth of web in inches.

„ t = Thickness of web in inches.

Then, $SF = f_s d_w t$; or, $f_s = \frac{SF}{d_w t}$; or, $t = \frac{SF}{f_s d_w}$.



WEB OF PLATE GIRDER STIFFENED BY MEANS OF TEE-IRON STIFFENERS.

In practice, the depth of the girder is usually decided upon when designing the girder, and it is frequently taken as $\frac{1}{8}$ to $\frac{1}{12}$ of the span. The

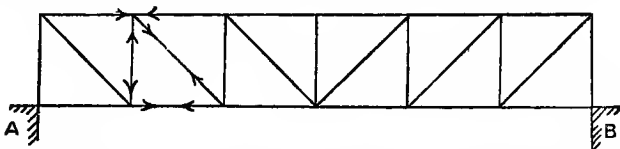
I am indebted to Prof. R. J. Woods, M.Inst.C.E., of Cooper's Hill College, and to Edward Arnold, publisher of his book on "Strength and Elasticity of Structural Members," for their kind permission to reproduce the six figures marked thus () on pp. 180 to 182, 186, 188.

thickness of the web is then determined, but should it, on calculation, come out less than $\frac{3}{8}$ inch, and it has been previously decided not to use a thinner web, then the depth in some instances is decreased accordingly within certain reasonable limits. The thickness of the web should, however, not be less than $\frac{3}{8}$ inch, to allow for deterioration, stiffness, and corrosion, due to damp, &c.

The shearing force increases from the middle of the span to the abutments, consequently, the thickness of the web plates may be reduced accordingly. But, it is sometimes more convenient to keep the web plate of the same thickness throughout.

We have seen, that the shear stress on a square element is equivalent to a tensile stress along one diagonal and a compressive stress along the other, each at 45° with the direction of the shear stress. Therefore, angle-iron or tee-iron stiffeners are usually riveted to the web at intervals approximately equal to the depth of the girder, for the purpose of preventing any buckling of the web due to the compressive stress.

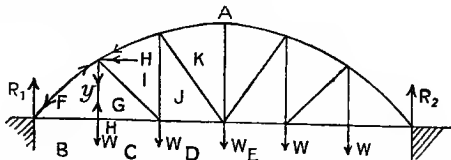
Function of the Web in Resisting the Shearing Force with Parallel Girders.—In the case of parallel girders, the stress on the flanges is horizontal; therefore, the vertical shearing force can only be resisted by the vertical component of the stresses in the diagonal bars or web, hence they are absolutely necessary in parallel girders. It is the horizontal component of these same stresses in the diagonal bars or web which produce the increment of flange stress at each panel point, as shown by the accompanying illustration.



STRESSES IN THE PANEL OF A LINVILLE OR N GIRDER.

Function of the Web in Resisting the Shearing Force with Parabolic Girders.—In the case of parabolic girders, the horizontal component of the stress at any point of the curved flange and the stress on the horizontal flange are constant.

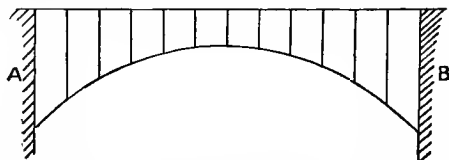
The bending moment at any section, whose co-ordinates are x and y , as in the accompanying figure, is $M = Hy$.



SHOWING FUNCTION OF THE WEB IN RESISTING THE SHEARING FORCE WITH PARABOLIC GIRDERS. (*)

The shearing force $SF = \frac{dM}{dx} = H \frac{dy}{dx} = H \tan i$, and the vertical component of the stress in the curved flange $V = H \tan i$.

Therefore, the vertical component suffices to balance the shearing force on the section. Consequently, no stress occurs in the diagonal web bracing of a parabolic girder when the girder is uniformly loaded with a dead load. Hence, in such a girder subject to a uniform dead load the diagonal bracing may be omitted.



PARABOLIC GIRDER WITH THE HORIZONTAL FLANGE
REPLACED BY ABUTMENTS. (*)

It will be seen, that the stress in the horizontal flange is entirely due to the thrust of the ends of the curved flange, and might, therefore, be replaced by abutments. This is then known as the principle of the *arch*.

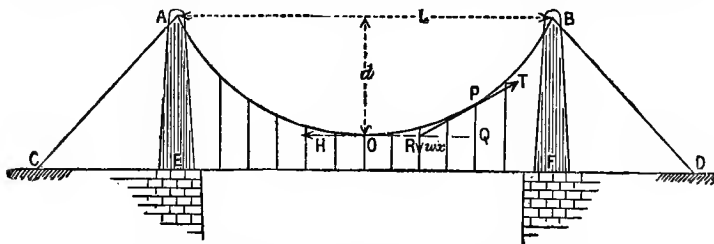
The above reasoning applies equally to the case where the bridge has the curved flange below the horizontal, except that the stresses are reversed, the horizontal flange being now in compression, and the curved flange in tension.



PARABOLIC GIRDER WITH CURVED FLANGE BELOW
THE HORIZONTAL. (*)

Suspension Bridges.—The horizontal flange in the previous figure can now be replaced by anchorages at the abutments, then it is known as the principle of the suspension bridge.

Definition of a Suspension Bridge.—A suspension bridge is one in which the platform is suspended by steel rods, from link or wire-rope cables, which pass over bearers on the towers erected at the piers and abutments.



SKETCH OF A SUSPENSION BRIDGE.

These cables or link chains are anchored down at their ends to the ground. When a flexible chain of uniform weight per unit of length is suspended at both ends, and allowed to hang freely, it takes the form of

a catenary curve (see Lect. I., Vol. II.). In practice, with uniformly loaded suspension bridges, the load is uniformly distributed with reference to a horizontal line, and is usually suspended from the cables by rods placed at equal distances apart. The curve which the chain or cable assumes when thus loaded, is a parabola. When a moving load passes over the bridge, the curve which the chains assume changes with each position of the load, if the bridge is not stiffened.

Parabolic Chains, or a Chain Uniformly Loaded per Foot Run of Span.

Let	A O B	represent the Chain	suspended at the points A and B.
„	w	„	Uniform load per unit length or foot run of span.
„	l	„	Length of span in feet.
„	d	„	Dip, versine, or depth of lowest point of the curve below the horizontal drawn through AB.
„	x and y	„	Co-ordinates of any point of the chain.
„	T	„	Tension at any point.
„	H_0	„	Horizontal component of tension on each chain or stress at the apex O of the parabola.

Taking the origin O at the lowest point of the chain, the total load borne by any arc OP is $w \times OP = wx$.

This load acts through R, the middle point of OQ. The tangent at P meets OQ in R, and consequently bisects OQ.

Hence, the portion OP of the chain is kept in equilibrium by (1) the weight of the chain wx , acting at R; (2) the tension at P, acting tangentially to the chain; (3) the horizontal tension H, acting at the origin O. Then, these three forces must meet in the same point R, and are represented by the triangle of forces PQR.

Therefore,
$$\frac{y}{\frac{1}{2}x} = \frac{wx}{H}; \text{ or, } y = \frac{wx^2}{2H}. \quad (1)$$

An equation which shows that the form of the chain is a parabola, with its vertex at O.

From equation (1),
$$H = \frac{wx^2}{2y}. \quad (2)$$

Then, the direct tension at P is equal to $PR = RQ \sec i$, where i is the angle which the tangent at P makes with the horizontal.

Or,
$$PR^2 = RQ^2 + QP^2.$$

Substituting values,
$$T^2 = H^2 + (wx)^2 = \frac{w^2 x^4}{4y^2} + w^2 x^2.$$

$$\therefore T = wx \sqrt{\frac{x^2}{4y^2} + 1}. \quad (3)$$

Hence, this last equation will give the tension at any point of the chain, and at the ends A and B, where the tension is greatest. That is, where $x = \frac{L}{2}$ and $y = d$, we get, by substituting these values in equations (2) and (3)—

$$H = \frac{w x^2}{2y} = \frac{w L^2}{8d}. \quad (4)$$

$$T = wx \sqrt{\frac{x^2}{4y^2} + 1} = \frac{wL}{2} \sqrt{\frac{L^2}{16d^2} + 1}.$$

Or,
$$T = \frac{wL}{8d} \sqrt{16d^2 + L^2}. \quad (5)$$

Method of Attaching the Main Chains to the Piers, and Finding the Pressure on the Piers in Suspension Bridges.—The previous figure represents a span, where AOB is called the main central chain or cable. AC and BD are the side or counter chains or backstays, which are anchored down to the ground at the points C and D; whilst EA and FB are the towers or piers.

There are two methods by which the chains may be carried over the piers :—

CASE 1.—The main and counter chains may be continuous in one length, and pass over smooth rounded saddles or pulleys fixed to the tops of the piers.

CASE 2.—The main and counter chains may be separate or independent chains, the ends of each being fastened to a saddle which is free to move horizontally on the top of the tower or pier.

In these two arrangements, the stresses on the backstays or counter chains, and the pressures on the piers differ considerably, as shown by the following analysis :—

Analytical Solutions for Tensions on Chains and Pressures on Piers of Suspension Bridges.

Let T_1 = Tension on the main central chain A B.

„ T_2 = Tension on the counter or backstay at B.

„ i_1 = Angle of inclination to the horizontal of the main chain at B.

„ i_2 = Angle of inclination to the horizontal of the backstay at B.

„ R = Vertical reaction or pressure on the pier.

CASE 1.—When the chain passes over the pulleys fixed on the top of the piers, the tensions T_1 and T_2 on the chains at each side of the pier will be the same, whether their angles of inclination i_1 , i_2 are the same or not.

When the angles of inclination are the same, the external pressure on the pier will be vertical. If the angles of inclination i_1 , i_2 be not equal then there will be a horizontal component to the pressure on the pier tending to turn it over in the direction of the chain whose angle of inclination to the horizontal is least.

Consequently, $R = T_1 (\sin i_1 + \sin i_2)$.

And there is a horizontal force at each pier—

$$H_P = T_1 (\cos i_1 - \cos i_2). \quad . \quad . \quad . \quad (6)$$

If $i_1 = i_2$, then, $R = 2 T_1 \sin i_1$. $. \quad . \quad . \quad . \quad (7)$

In this case, there is no horizontal force. Therefore, the vertical pressure on each pier is equal to twice the weight of the portion of the bridge between the pier and the lowest point of the curve, because T_1 was shown by equation (3) to be equal to the stress at the apex of the parabola into $\sec i$, and by substituting this value in the above equation (7), we get—

$$R = 2 T_1 \sin i = 2 H_0 \sec i \sin i = 2 H_0 \tan i = w L.$$

CASE 2.—When the ends of the main central chain and counter chain are fastened to the pier, or to a saddle which is free to slide horizontally on the top of the pier, the resultant pressure on the pier will always be vertical, no matter what the inclinations of the chains are to the horizontal. The tension on both chains will be the same when their angles of inclination are the same, but will be different when the angles vary, although the horizontal components of the tensions will be the same.

But, $T_1 = H_0 \sec i_1$ and $T_2 = H_0 \sec i_2$.

Therefore, the resultant vertical pressure on the pier will be—

$$R = T_1 \sin i_1 + T_2 \sin i_2 = H_0 (\tan i_1 + \tan i_2). \quad (8)$$

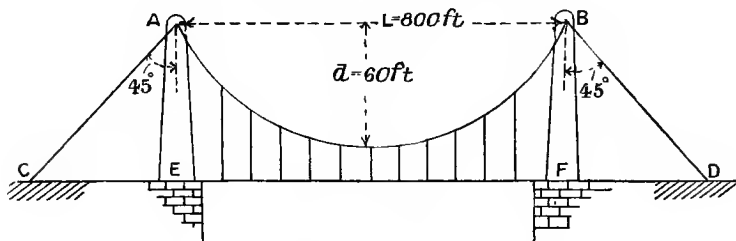
EXAMPLE VIII.—A suspension bridge has a span of 800 feet, a dip (or sag) of cord of 60 feet, and carries, by means of four cables, a total fixed load of 250 tons uniformly distributed along the length of the platform. Assuming the hanging rods to be very numerous, determine the tension in each cable at the lowest point and at the piers. Assuming the cables to be attached to saddles resting upon rollers on the tops of the piers and the anchor cables to make an angle of 45° with the vertical, determine the maximum stress in the anchor cables and the total pressure on each of the piers. (I.C.E., Feb., 1898.)

Total load $W = wL = 250$ tons.

Each cable carries $\frac{250}{4} = 62.5$ tons.

Let H_0 = the horizontal component of the tension in the cable, we have from equation (4)—

$$H_0 = \frac{wL^2}{8d} = \frac{WL}{8d} = \frac{62.5 \times 800}{8 \times 60} = 104\frac{1}{3} \text{ tons, tension at lowest point.}$$



SUSPENSION BRIDGE.

The vertical component of tension in central cable at either pier = $\frac{1}{2}$ load on cable = $\frac{62.5}{2} = 31.25$ tons.

$\therefore$ The resultant tension of the cable at the pier is obtained from equation (5)—

$$T = \frac{wL}{8d} \sqrt{16d^2 + L^2}.$$

$$T = \frac{62.5}{8 \times 60} \sqrt{16 \times (60)^2 + (800)^2} = \frac{25}{192} \sqrt{57,600 + 640,000}.$$

$$T = \frac{25}{192} \sqrt{697,600} = \frac{25 \times 835}{192} = 108.8 \text{ tons.}$$

The horizontal component of each anchor or backstay cable must equal $H_0 = 104\frac{1}{3}$ tons, and as it is inclined at 45° , the vertical component must also equal $104\frac{1}{3}$ tons.

Hence, total maximum tension acting along the backstay cables is obtained from equation (3)—

$$T = H_0 \sec i_2 = 104\frac{1}{2} \times \sec 45^\circ = 104\frac{1}{2} \times \sqrt{2} = 147.2 \text{ tons.}$$

Vertical pressure on each pier due to each cable is obtained from equation (6)—

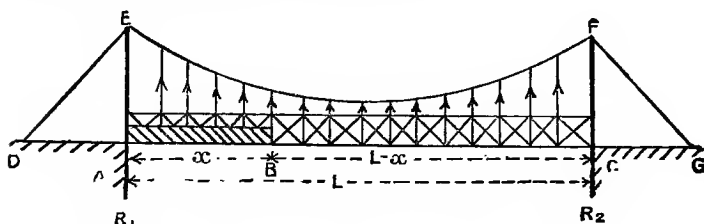
$$R = T_1 \sin i_1 + T_2 \sin i_2.$$

R = Vertical component of tension in central cable at the pier + vertical component of the stress in each backstay cable.

$$R = \frac{62.5}{2} + 104\frac{1}{2} = 31\frac{1}{2} + 104\frac{1}{2} = 135\frac{5}{8} \text{ tons.}$$

Therefore, the total vertical pressure on each pier which is due to the four cables = $4R = 4 \times 135\frac{5}{8} = 542 \text{ tons.}$

Stiffening Suspension Bridges by Means of Girders.—When this bridge is liable to heavy rolling loads passing over it, the shape of the cables become deformed. The lateral oscillations may be retarded by efficiently bracing the platform with horizontal and diagonal bracing bars. Whilst, another method is to introduce into the structure a pair of light lattice girders. These girders, in addition to stiffening the bridge, act the part of side parapets or screens. The girders may be hung from the main central chains by aid of the vertical suspending rods, and the cross-beams of the bridge attached to their bottom flanges. The ends of these girders are fastened down to the piers to prevent them being lifted when the load is passing over them. The object for the stiffening girders is to distribute the load uniformly over the cables, and thus prevent the latter becoming distorted. The booms and bracing bars of the girder should be designed for taking either a tensile or compressive stress.



SUSPENSION BRIDGE STIFFENED BY MEANS OF GIRDERS. (*)

Single Girder Continuous from Pier to Pier under a Uniform Live Load.—*Reactions.*—The moment the live load comes on to the suspension bridge, the stiffening girders distribute the load uniformly to the cables. Should the load be small in comparison with the weight of the cables, then the cables will keep their parabolic curve, and the stresses in the suspending rods will all be equal.

In the accompanying figure, A and C are the supports or piers; and, let us take it, that the girder is loaded over a portion x , with a live load of intensity w tons per foot run.

Then, if P = Pull on each suspending rod in tons.

p = Uniform upward pull of the suspending rods per foot run.

L = Length or span of girder in feet.

R_1, R_2 = Reactions due to the part of the load.

And, assuming that the weight is transmitted through the suspending rods, we have—

$$pL = wx. \quad . \quad p = \frac{wx}{L}. \quad . \quad . \quad . \quad (1)$$

Now, considering the conditions of equilibrium, we get—

$$R_1 + R_2 + pL - wx = 0. \quad \text{But, } pL = wx.$$

$$\therefore R_1 + R_2 = 0.$$

Taking moments around the point C—

$$R_1L + \frac{pL^2}{2} - \frac{wx^2}{2} = 0.$$

$$\therefore R_1 = -\left(\frac{pL^2}{2} - \frac{wx^2}{2}\right) = -\frac{wx}{2L}(L-x).$$

$$\text{Hence, } R_2 = -R_1 = \frac{wx}{2L}(L-x). \quad . \quad . \quad . \quad . \quad (2)$$

Therefore, the reactions are equal and opposite, and will be a maximum when $x = \frac{L}{2}$; giving their maximum value equal to $\frac{wL^2}{8}$. Consequently, the maximum shearing force occurs at the supports when the live load covers half the span; and the magnitude of the shearing force at the end of the live load on the bridge is equal to that of either reaction.

Shearing Forces.—The magnitude of the shearing force in the loaded part of the suspension bridge at a distance x_1 from the left-hand support will be—

$$SF = R_2 + px_1 - wx_1.$$

Substituting the values for R_2 and p from equations (1) and (2)—

$$SF = \frac{wx}{2L}(L-x) + \frac{wx_1x}{L} - wx_1.$$

$$SF = \frac{wxL - wx^2 + 2wx_1x - 2wLx_1}{2L} = \frac{w(L-x)}{L}\left(\frac{x}{2} - x_1\right). \quad (3)$$

Hence, if $x_1 = \frac{x}{2}$, then the shearing force $SF = 0$, and it can easily be shown that the shearing force is zero at the middle of the unloaded part of the suspension bridge. Also, from equation (3) we see that if $x_1 = x$, then the SF is equal to $-R_2$.

Bending Moments.—The bending moment at any section of the loaded part of the bridge distant x_1 , say, from the left-hand support, is—

$$BM = R_2x_1 + \frac{px_1^2}{2} - \frac{wx_1^2}{2}.$$

$$BM = \frac{wx}{2L}(L-x)x_1 + \frac{wx_1^2}{2L} - \frac{wx_1^2}{2}.$$

$$BM = \frac{wxLx_1 - wx^2x_1 + \frac{wx_1^2}{L} - \frac{wx_1^2}{2}}{2L} = \frac{w(L-x)}{2L}(xx_1 - x_1^2). \quad (4)$$

From this equation we see that the bending moment $BM = 0$, when $x_1 = x$, also when $x_1 = 0$.

The maximum bending moment occurs when $x_1 = \frac{x}{2}$, that is where the shearing force is nothing.

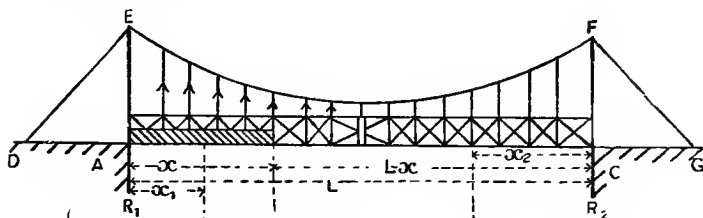
$$\text{Max. } BM = \frac{w(L-x)}{8L} x^2. \quad . \quad . \quad . \quad (5)$$

Greatest Bending Moment for a Moving Load.—To find the greatest BM as the load advances across the bridge, we equate $\frac{d}{dx}(Lx^2 - x^3)$ to zero, from which we get $x = \frac{2}{3}L$.

$$\text{Therefore, Greatest } BM = \frac{w(L - \frac{2}{3}L)}{8L} (\frac{2}{3}L)^2 = \frac{wL^2}{54}. \quad . \quad (6)$$

This maximum bending moment will take place when the live load covers two-thirds of the span.

If we consider the unloaded portion of the bridge, then the maximum bending moment also takes place at its middle point. Its maximum value occurs when the live load has travelled over one-third of the span, and is equal to $-\frac{wL^2}{54}$.



SUSPENSION BRIDGE WITH THE STIFFENING GIRDER HINGED AT THE CENTRE. (*)

Suspension Bridges with the Stiffening Girder Hinged at the Centre.—Professor Rankine† says, “In order to enable it (the suspension bridge) to act with the greatest efficiency, it should be *hinged* at the middle of the span, which may be effected by making it in two halves, connected together by means of a cylindrical pin of dimensions sufficient to bear the shearing stress,” which will presently be stated. The object of the hinge at the centre is to annul the straining action which would arise from the deflection and expansion of the chain. It provides for contraction and expansion due to the changes of temperature.

The cable or central chain is assumed to retain its parabolic shape when the span is partially loaded, in the same way, as assumed in the previous case. Therefore, all the suspending rods are subjected to an equal stress. There will be no bending moment at the centre of the span, owing to the hinge being placed there.

† See Rankine's *Civil Engineering*, p. 579.

Using the same notation as given in the previous case, and taking the live load as occupying a portion of the bridge distant x feet from the left-hand support.

Applying the conditions of equilibrium, we have as before—

$$R_1 + R_2 + pL - wx = 0. \quad \therefore R_1 + R_2 = 0. \quad (1)$$

Taking moments about the hinge in the centre of the bridge, we get—

$$R_2 \frac{L}{2} + p \frac{L^2}{8} - \frac{wx}{2} (L - x) = 0. \quad (2)$$

And,
$$R_1 \frac{L}{2} + \frac{pL^2}{8} = 0. \quad (3)$$

Consequently, from these three equations, we get—

$$p = \frac{2wx^2}{L^2}. \quad (4)$$

$$R_2 = \frac{w}{2L} (2Lx - 3x^2). \quad (5)$$

$$R_1 = -\frac{w}{2L} x^2. \quad (6)$$

Hence, R_2 will be a maximum when $x = \frac{L}{3}$; and max. $R_2 = \frac{wL}{6}$.

$$R_1 \quad ,, \quad ,, \quad x = \frac{L}{2}; \quad ,, \quad R_1 = -\frac{wL}{8}.$$

The magnitude of the shearing force at the end of the load on the bridge will be—

$$SF = R_2 + px - wx = \frac{w}{2L^2} (4x^3 - 3x^2L). \quad (7)$$

This equation becomes a maximum when $x = \frac{L}{2}$, and is equal to $\frac{wL}{8}$.

The maximum bending moment occurs at the section where the shearing force is nothing.

To find the magnitude of both the bending moment and shearing force at any point of the loaded portion of the girder, distant, say, x_1 feet from the left-hand support, we have—

$$BM = R_2 x_1 + \frac{p x_1^2}{2} - \frac{w x_1^2}{2}. \quad (8)$$

$$SF = R_2 + p x_1 - w x_1. \quad (9)$$

If the $SF = 0$, then from equation (9)—

$$SF = R_2 + p x_1 - w x_1 = 0.$$

$$x_1 (w - p) = R_2.$$

$$\therefore x_1 = \frac{R_2}{w - p} = \frac{(2Lx - 3x^2)L}{2(L^2 - 2x^2)}.$$

Also, the bending moment from equation (8) will be—

$$BM = R_2 x_1 + \frac{p x_1^2}{2} - \frac{w x_1^2}{2} = \frac{1}{2} \frac{R_2^2}{w - p} = \frac{w (2Lx - 3x^2)^2}{8(L^2 - 2x^2)}. \quad (10)$$

Hence, differentiating and equating to zero for the max. BM , we get—

$$(L^2 - 2x^2)(L - 3x) + x^2(2L - 3x) = 0.$$

Or,
$$3x^3 - 3L^2x + L^3 = 0.$$

$\therefore x = .4L$, approximately.

Substituting this value for x in equation (10), we obtain the magnitude of the maximum positive bending moment.

$$B M = \frac{w (2 L x - 3 x^2)^2}{8 (L^2 - 2 x^2)} = \frac{w L^2}{53}.$$

For the right-hand half of the span of the suspension bridge, at a point distant, say, x_2 feet from the right-hand tower, we have—

$$S F = R_1 + p x_2 = - \frac{w x^2}{2 L} + \frac{2 w x^2}{L^2} x^2. \quad (11)$$

$$B M = R_2 x_2 + \frac{p x_2^2}{2}. \quad (12)$$

On examining equation (11) we find that when $S F = 0$, $x_2 = \frac{l}{4}$. and the bending moment $B M = - \frac{w x^2}{16}$.

Therefore, maximum negative bending moment,

$$B M = - \frac{w x^2}{16} = - \frac{w L^2}{4 \times 16} = - \frac{w L^2}{64}.$$

LECTURE VIII.—QUESTIONS.

1. Let the ends of each of the bays of the roof truss in Fig. 49, Lecture III., Vol. III., be fixed so that a load of 1 ton is acting at each of the joints in the rafters; draw a frame and a stress diagram to scale, tabulating the stresses in the various members in the manner indicated by the table on p. 39. Express your own views regarding this kind of roof for large spans as compared with any other style.

2. Draw and explain different methods of constructing the parabola upon a given base.

3. Explain Culman's and Ritter's methods of sections by means of sketches, &c.

4. Explain concisely and clearly, by aid of scale drawings, how you would determine graphically the maxima and minima stresses in the members of a *Curved Trussed Girder* due to a live load.

5. Explain the functions of the web in resisting the shearing force with parallel and with parabolic girders.

N.B.—See *Appendices B and C* for other questions and answers.

APPENDICES.

APPENDIX A (p. 194 to p. 199).

- (i.) General Instructions by The Board of Education for their Examinations on Applied Mechanics.
- (ii.) General Instructions by The City and Guilds of London Institute for their Examination on Mechanical Engineering.
- (iii.) Rules and Syllabus of Examinations by The Institution of Civil Engineers for Election of Associate Members.

APPENDIX B (p. 200 to p. 217).

- (i.) Board of Education's Exam. Papers in Applied Mechanics, Stages 2 and 3. The City and Guilds of London Institute's Honours Exam. Papers in Mechanical Engineering. And, The Institution of Civil Engineers' Exam. Papers in Theory of Structures, arranged in the order of the Lectures.
- (ii.) All new Answers to Questions for the respective Lectures are tabulated under the two main headings—*either* Board of Education and City and Guilds *or* Institution of Civil Engineers.

APPENDIX C (p. 218 to p. 220).

The latest Exam. Papers pertaining to Theory of Structures and set by the governing bodies enumerated under Appendix A.

APPENDIX D (p. 221 to p. 226).

Tables of Constants, Logarithms, Antilogarithms, and Functions of Angles.

APPENDIX A.

MAY EXAMINATION ON SUBJECT VIIA. APPLIED MECHANICS.*

BY THE BOARD OF EDUCATION SECONDARY BRANCH,
SOUTH KENSINGTON, LONDON.

GENERAL INSTRUCTIONS.

If the rules are not attended to, your paper will be cancelled.

Immediately before the Examination commences, the following
REGULATIONS are TO BE READ TO THE
CANDIDATES.

Before commencing your work, you are required to fill up the numbered slip which is attached to the blank examination paper.

You may not have with you any books, notes, or scribbling paper.

You are not allowed to write or make any marks upon your paper of questions, or to take it away before the close of the examination.

You must not, under any circumstances whatever, speak to or communicate with one another, and no explanation of the subject of examination may be asked or given.

You must remain seated until your papers have been collected, and then quietly leave the examination room. None of you will be permitted to leave before the expiration of one hour from the commencement of the examination, and no one can be re-admitted after having once left the room.

Your papers, unless previously given up, will all be collected at 10 o'clock.

If any of you break any of these rules, or use any unfair means, you will be expelled, and your paper cancelled.

**Before commencing your work, you must carefully
read the following instructions:—**

Candidates who have applied for examination in the Elementary Stage must confine themselves to that stage. Candidates who have not applied to take the Elementary Stage may take Stage 2, or Stage 3, or, if eligible, Honours, but they must confine themselves to one of them.

Put the number of the question before your answer.

You are to confine your answers *strictly* to the questions proposed.

Such details of your calculations should be given as will show the methods employed in obtaining arithmetical results.

A table of logarithms and functions of angles and useful constants and formulæ is supplied to each candidate. (See end of Appendix to this Book.)

The examination in this subject lasts for three hours.

* See Appendix C for the latest Exam. Papers.

City and Guilds of London Institute.

DEPARTMENT OF TECHNOLOGY.

TECHNOLOGICAL EXAMINATIONS.

46.—MECHANICAL ENGINEERING.*

HONOURS GRADE (Written Examination).

INSTRUCTIONS.

The Candidate for Honours must have previously passed in the Ordinary Grade, and is required to pass a Written and Practical Examination. He is requested to state, on the Yellow Form, whether he has elected to be examined in A, Machine Designing, or in B, Workshop Practice (*a*) *Fitting*, (*b*) *Turning*, (*c*) *Pattern-making*. Candidates in Machine Designing must forward their work to London not later than May 6th,† and in Workshop Practice not later than May 13th.†

The number of the question must be placed before the answer in the worked paper.

The Candidate is at liberty to use divided scales, compasses, set squares, calculators, slide rules, and mathematical tables.

Five marks extra will be awarded for every answer worked out with the slide rule, provided the method of working is explained.

The maximum number of marks obtainable is affixed to each question.

Three hours allowed for this paper.

The Candidate is not expected to answer more than *eight* of the following questions, which must be selected from *two* sections *only*.

* The questions in Sections A and C may be answered from Vols. I. to V. of my *Text-Book on Applied Mechanics*. Section B is printed in my *Text-Book on Steam and Steam Engines*. The Questions for the Ordinary Grade are printed at the end of my *Elementary Manual on Applied Mechanics*.

* See Appendix C for the latest Exam. Papers.

† These dates are only approximate, and subject to a slight alteration each year.

The Institution of Civil Engineers.

EXTRACTS FROM RULES AND SYLLABUS OF EXAMINATIONS FOR ELECTION OF ASSOCIATE MEMBERS.

Note.—Engineers who desire to enter for the A.M.Inst.C.E. examinations should write *at once* to the Secretary, Great George Street, Westminster, S.W., for the complete Rules, Syllabus, and Application Forms. They will find all the questions relating to the above mentioned subjects which have been set since these examinations commenced in 1897 in my **Text-Books**.

The following extracts are simply printed here to show how far my books upon *Applied Mechanics and Mechanical Engineering*; *Steam and Steam Engines, including Turbines and Boilers*; as well as *Magnetism and Electricity* (including Munro & Jamieson's *Pocket-Book of Electrical Rules and Tables*), together with my "*Correspondence System of Electrical and Mechanical Engineering Science, as taught by Exercises, Drawings, and Instructions*" cover the Scientific and Practical Knowledge demanded by the Institution, under Part II., Section A (1, 2, 3a, and 3b), as well as Section B under Group i., Theory of Heat Engines; Group ii., Hydraulics and Theory of Machines; and Group iii., Applications of Electricity.

Note for Students.—In the regulations for Students one of the subjects which may be selected is that of Elementary Mechanics of Solids and Fluids, for which see my *Elementary Applied Mechanics* when studying this subject. Also, see my *Elementary Magnetism and Electricity* book when reading that part of the Elementary Physics for admission of Students, or correspond with me re those two subjects.

PART II.*—Scientific Knowledge.

SECTION A.

1. Applied Mechanics (one Paper, *time allowed, 3 hours*).
2. Strength and Elasticity of Materials (one Paper, *time allowed, 3 hours*).

* Candidates may offer themselves for examination in Sections A and B of Part II. together; or they may enter for Section A alone, and, if successful, may take Section B at a subsequent examination. In the latter case, however, such candidates will not be allowed to present themselves for examination in Section B unless or until they are actually occupied in work as pupils or assistants to practising Engineers. The Council may permit Candidates who have attempted the whole of Part II. at one examination, and have failed in Section B only, to complete their qualification by passing in that section at a subsequent examination, subject to their being then occupied as above stated.

3. *Either* (a) **Theory of Structures.**
 or (b) **Theory of Electricity and Magnetism** (one Paper,
time allowed, 3 hours).

SECTION B.

Two of the following nine subjects—not more than one from any group (one Paper in each subject taken, *time allowed, 3 hours for each Paper*) :—

Group i.	Group ii.	Group iii.
Geodesy.	Hydraulics.	Geology and Mineralogy.
Theory of Heat	Theory of Machines.	Stability and Resistance
Engines.	Thermo- and Electro-	of Ships.
Metallurgy.	Chemistry.	Applications of Elec- tricity.

Mathematics.—The standard of Mathematics required for the Papers in Part II. of the examination is that of the mathematical portion of the Examination for the Admission of Students, though questions may be set involving the use of higher Mathematics.

The range of the examinations in the several subjects, in each of which a choice of questions will be allowed, is indicated generally hereunder :—

SECTION A.

1. Applied Mechanics :—

Statics.—Forces acting on a rigid body ; moments of forces, composition, and resolution of forces ; couples, conditions of equilibrium, with application to loaded structures. The foregoing subjects to be treated both graphically and by aid of algebra and geometry.

Hydrostatics.—Pressure at any point in a gravitating liquid ; centre of pressure on immersed plane areas ; specific gravity.

Kinematics of Plane Motion.—Velocity and acceleration of a point ; instantaneous centre of a moving body.

Kinetics of Plane Motion.—Force, mass, momentum, moment of momentum, work, energy, their relation and their measure ; equations of motion of a particle ; rectilinear motion under the action of gravity ; falling bodies and motion on an inclined plane ; motion in a circle ; centres of mass and moments of inertia ; rotation of a rigid body about a fixed axis ; conservation of energy.

2. Strength and Elasticity of Materials :—

Coefficients of elasticity ; elastic resistance to tension, compression, shearing, and torsion ; uniform and varying stress ; moment of stress and of resistance in beams of various sections ; distribution of shearing stress in beams ; stresses suddenly applied and effects of impact ; buckling of struts ; effect of different end-fastenings on their resistance ; combined strains ; calculations connected with statically indeterminate problems, as beams supported at three points, &c. ; limit of elasticity, yield-point, and

ultimate resistance of various materials as tested: the plastic state; principal forms of testing machines and of appliances used in measurements for the determination of coefficients of elasticity; calculation of extension, deflection, buckling, &c., within elastic limits, and of ultimate strength and ductility for ordinary materials of construction.

3. (a) Theory of Structures:—

Graphic and analytic methods for the calculation of bending moments and of shearing forces, and of the stresses in individual members of framework structures loaded at the joints; plate and box girders; incomplete and redundant frames; theory of continuous girders and principal methods of calculation; travelling loads; riveted and pin-joint girders; rigid and hinged arches; strains due to weight of structures; theory of earth-pressure and of foundations; stability of masonry and brickwork structures.

3. (b) Theory of Electricity and Magnetism:—

Electrical and magnetic laws, units, standards, and measurements; electrical and magnetic measuring instruments; the theory of the generation, storage, transformation, and distribution of electrical energy; continuous and alternating currents; arc and incandescent lamps; secondary cells.

SECTION B.

Group i. Theory of Heat Engines:—

Thermodynamic laws; internal and external work; graphical representation of changes in the condition of a fluid; theory of heat engines working with a perfect gas; air- and gas-engine cycles; reversibility, conditions necessary for maximum possible efficiency in any cycle; properties of steam; the Carnot and Clausius cycles; entropy and entropy-temperature diagrams, and their application in the study of heat engines; actual heat engine cycles and their thermodynamic losses; effects of clearance and throttling; initial condensation; testing of heat engines, and the apparatus employed; performances of typical engines of different classes; efficiency.

Group ii. Hydraulics:—

The laws of the flow of water by orifices, notches, and weirs; laws of fluid friction; steady flow in pipes or channels of uniform section; resistance of valves and bends; general phenomena of flow in rivers; methods of determining the discharge of streams; tidal action; generation and effect of waves; impulse and reaction of jets of water; transmission of energy by fluids; principles of machines acting by the weight, pressure, and kinetic energy of water; theory and structure of turbines and pumps.

Theory of Machines:—

Kinematics of machines; inversion of kinematic chains; virtual centres; belt, rope, chain, toothed and screw gearing; velocity, acceleration

and effort diagrams; inertia of reciprocating parts; elementary cases of balancing; governors and flywheels; friction and efficiency; strength and proportions of machine parts in simple cases.

Group iii. Applications of Electricity:—

Theory and design of continuous- and alternating-current generators and motors, synchronous and induction motors and static transformers; design of generating- and sub-stations and the principal plant required in them; the principal systems of distributing electrical energy, including the arrangement of mains and feeders; estimation of losses and of efficiency; principal systems of electric traction; construction and efficiency of the principal types of electric lamps.

43 Candidates should see, that all their "Forms" are duly completed and passed by the Council of the Institution of Civil Engineers, Great George Street, Westminster, S.W., before 1st January for the February Examination, and before the 1st September for the October Examination. Candidates should, therefore, apply to the Secretary for the "Forms," at least six months before these Examinations, to give them time to make due and proper Application, and to thoroughly *Revise* the subjects upon which they are to be examined with an experienced **Guide** and Tuition by Correspondence.

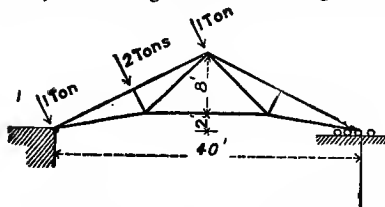
Examinations Abroad.—The papers of the October Examination only will be placed before accepted Candidates in India and the Colonies. To enable the Secretary to make arrangements for the Application Forms and Fees, &c., of these Candidates, their Forms, &c., must be in the Secretary's hands, before the 1st June preceding the October Examinations.

N.B.—See *Appendix C* for the latest *Exam. Papers in Theory of Structures*.

APPENDIX B.

LECTURE II.—ORDINARY QUESTIONS.

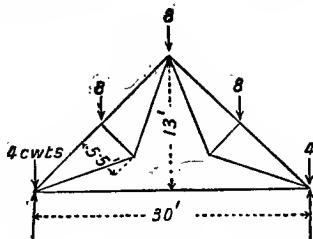
1. Draw the force diagram for the roof shown in the figure, the wind blowing from the left, and the right-hand end being on rollers.



(C. & G., H., Sec. A, 1906.)

2. Draw the force diagram for the roof truss shown, and tabulate the forces in the different members.

(C. & G., H., Sec. A, 1907.)



3. In the case of large steel roofs, as, for example, station roofs, describe two methods for providing against the expansion of the steel. Taking one of these cases, indicate, in general terms, how you would proceed to draw the force diagram—vertical loads only being considered.

(C. & G., H., Sec. A, 1907.)

LECTURE IV.—ORDINARY QUESTIONS.

1. The legs of a tripod are 15, 16.5, and 18 feet long respectively, and the lengths of the lines joining the feet of the legs are each 20 feet. Find the force along each leg when a weight of 10 tons is suspended from the apex. Graphical constructions may be used. (C. & G., H., Sec. A, 1903.)

2. A tripod has the following dimensions:—The apex point is O, and the lengths of the three legs AO, BO, and CO are respectively 18 feet, 17.5 feet, and 16 feet. The lengths of the sides of the triangle formed by the feet AB, BC, and CA are 9 feet, 9.5 feet, and 10 feet respectively.

Find graphically, or in any other way, the forces which act down each leg of the tripod when a load of 10 tons is suspended from it.

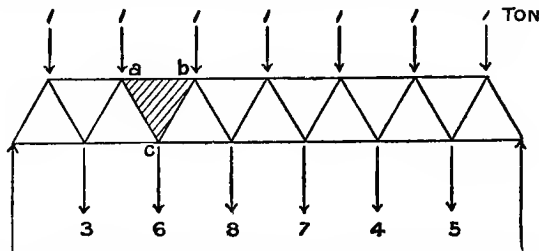
(B. of E., S. 3, 1906.)

LECTURE VIII.—ORDINARY QUESTIONS.

1. What are the functions of the top and bottom booms, and of the diagonal pieces of a railway girder? Why are the booms usually larger in section towards the middle of the girder, and the diagonal pieces larger towards the ends of the girder? (B. of E., Adv., 1903.)

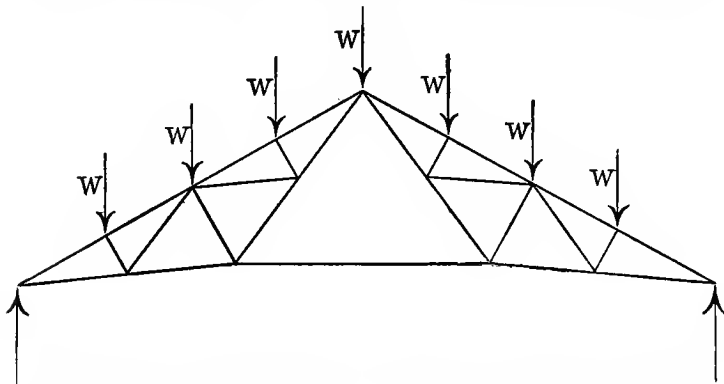
2. A heavy chain is supported by its ends, A and B, which are 12 feet above the lowest part of the chain. The horizontal distance between A and B is 66 feet, and the weight of the chain is 20 lbs. per foot of its horizontal projection. Draw out to scale (10 feet to the inch) the shape of the chain, and find the force in the chain at the lowest point. What is the maximum force in the chain? (B. of E., Adv., 1903.)

3. Explain what is meant by the "method of sections" in determining the forces in the members of a braced structure. Find, by this means, the



forces in the members ab , bc , and ca , in the Warren girder shown in the figure. (C. & G., H., Sec. A, 1904.)

4. A roof truss, subject to vertical loads, is as sketched. Explain fully



how the force in each member may be obtained. You may adopt either graphical or analytical solutions. (C. & G., H., Sec. A, 1905.)

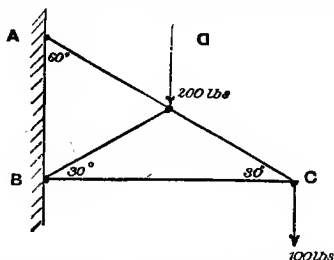
5. Explain how to determine the stresses in all the members of a frame by the method of sections. A girder 120 feet long, with parallel chords 16 feet apart, has eight panels of **N** form, and is loaded with 20 tons at each of the top joints. Calculate the stresses in all the members of the second and third panels from one end. (C. & G., 1908, H., Sec. A.)

LECTURE I.—I.C.E. QUESTIONS.

1. A string, $A B C D$, hangs in a vertical plane, the ends A and D being fixed. A weight of 10 lbs. is hung from the point B and an unknown weight from the point C . The middle portion $B C$ is horizontal and the portions $A B$ and $C D$ are inclined at 30° and 45° to the horizontal respectively. Determine the unknown weight and the tensions in the three portions of the string. (I.C.E., Oct., 1903.)

2. In respect to structures, what is meant by the terms "redundant," "deficient," and "counter-bracing"? Give an example of each. How would you ascertain whether a frame is "redundant," "deficient," or "simply firm"? (I.C.E., Oct., 1903 and 1905.)

3. A braced cantilever is as shown in figure. Find the forces acting in



each member, and distinguish between ties and struts. (I.C.E., Feb., 1904.)

4. A masonry dam, 20 feet high, 4 feet wide at the top and 12 feet wide at the bottom, has its faces straight, the water face being vertical. Draw the line of thrust when the water-level is 2 feet from the top. Take the specific gravity of the masonry as 2.25. (I.C.E., Feb., 1904.)

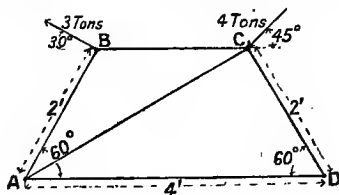
5. A small masonry dam is to be built of rectangular section with vertical faces. The specific gravity of the masonry will be 2.0, and the wall will stand 12 feet high, its top coinciding with the water-level. If the line of pressure at the base is not to stray beyond the middle third, what must be the thickness of the wall? (I.C.E., Feb., 1905.)

6. The wall of a water-tank is 7 feet deep, the section being rectangular, and the weight of the masonry 120 lbs. per cubic foot. Neglecting all forces except the weight of the wall and the pressure of the water, find the thickness of the wall in order that the line of action of the resultant pressure shall pass through the extremity of the middle third of the base joint, when the depth of the water is 6 feet. (I.C.E., Oct., 1905.)

7. A tramcar weighing 8 tons is moving round a curve of 50 feet radius at 8 miles per hour, the plane of the rails being horizontal and the gauge 3 feet 6 inches. If the surface of the car is taken as equivalent to a rectangle 15 feet high and 20 feet long and its centre of gravity 5 feet above the rails, find the least normal wind-pressure which would overturn the car. (I.C.E., Feb., 1906.)

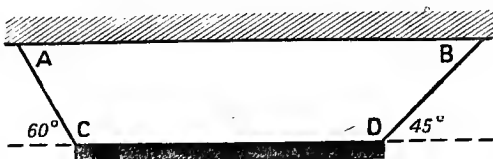
8. The simple frame is loaded at B and C as shown. It is supported by a vertical reaction at A and by a reaction at D . Find the magnitude

of the reaction at A, the magnitude and direction of the reaction at D, and the forces in the bars, stating which are struts and which are ties.



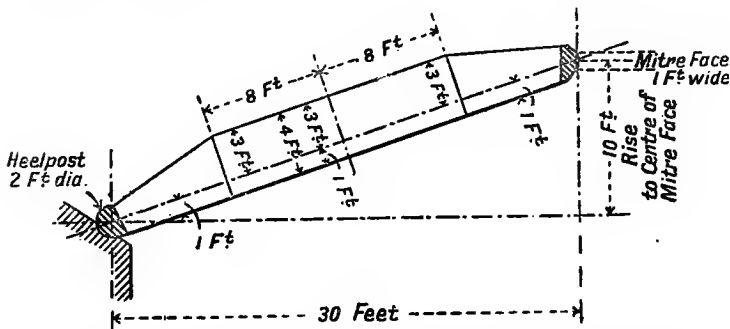
(I.C.E., Oct., 1907.)

9. A uniform beam CD, 6 feet long, carrying a weight of 1,000 lbs., is in a horizontal position when suspended by two ropes CA and DB at the angles shown in the figure. Find the position of the weight, (a) if the



weight of the beam is neglected, (b) if the weight of the beam is 500 lbs. In case (a) find the tensions in the ropes. (I.C.E., Feb., 1908.)

10. One leaf of a pair of dock gates has the form and dimensions in plan as shown. Determine the directions and amounts of the reactions at the



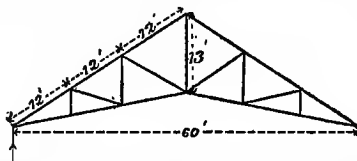
centre of the heel-post and the centre of the face of the mitre-post for a rib 2 feet in vertical depth under an average head of 20 feet of sea-water. Determine also the position, direction, and amount of the resultant thrust on the section at the centre of length of the leaf, midway between heel-post and mitre-post. Sea water weighs 64 lbs. per cubic foot.

(I.C.E., Oct., 1908.)

LECTURE III.—I.C.E. QUESTIONS.

1. A roof truss consists of two rafters, equally inclined, connected by a horizontal tie at a distance of a feet below the apex of the truss. If the span be l feet and the load at the apex of the truss w tons, show that the tension in the tie-rod is $\frac{wl}{4a}$ tons. (I.C.E., Oct., 1903.)

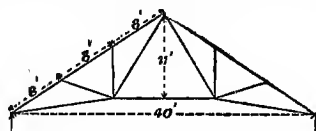
2. In the roof frame sketched in the figure, the left supporting force is vertical. The dead load being 50 lbs. per foot run of slope, and the



normal wind pressure on the left slope being 100 lbs. per foot run, find the stresses in the members, showing which are ties and which are struts.

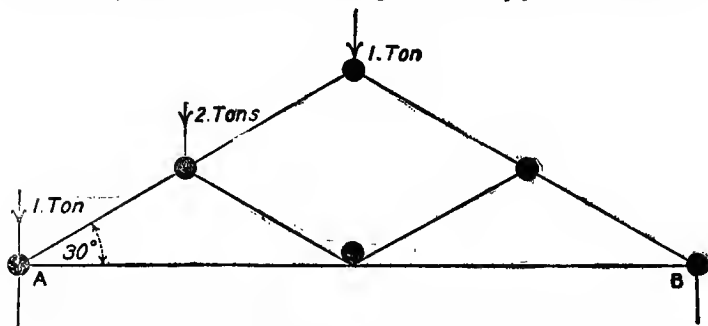
(I.C.E., Oct., 1903.)

3. Find the stresses in the members of the roof frame shown in the sketch, and mark the stress beside each member, + representing tension,



and - compression. The dead load is 100 lbs., and the normal wind pressure 150 lbs. per foot run of slope. The wind is on the right slope) and the left reaction is vertical. (I.C.E., Feb., 1904.)

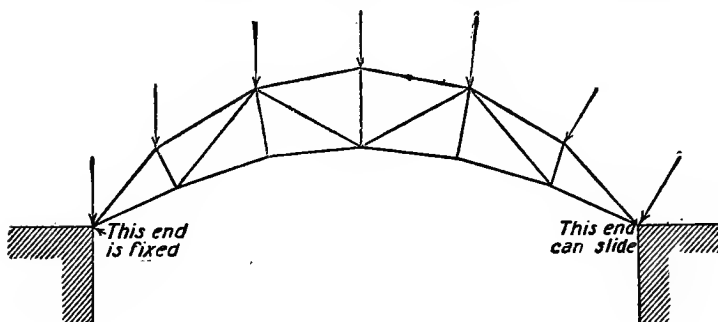
4. In the roof truss shown in the figure, the tie-bar AB is continuous from A to B, the rafters and struts being connected by joints as indicated.



Find by calculation, or graphically, the stresses in all the members and sketch the curves of shearing force and bending moment for the bar AB.

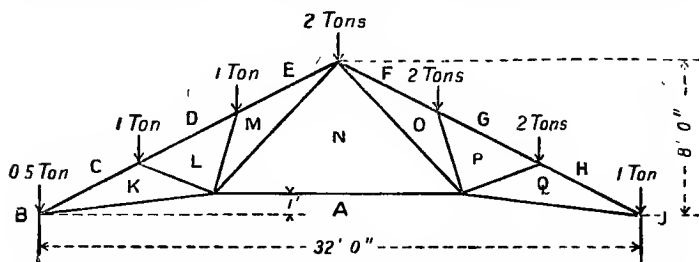
(I.C.E., Feb., 1906.)

5. Draw the stress diagram for the frame shown in the accompanying figure, assuming the joints to be loaded with equal weights in the



direction of the arrows. Indicate which bars are in tension and which are in compression. (I.C.E., Feb., 1906.)

6. Find the stresses in the various members of the roof truss shown in



the figure, indicating in tabular form which members are in tension and which in compression. (I.C.E., Feb., 1908.)

LECTURE IV.—I.C.E. QUESTIONS.

1. A pit crane is as sketched—*a* being a footstep bearing and the reaction at *b* assumed horizontal. Find, graphically or otherwise, the forces in the different members when a load of 10 tons is suspended from the extremity of the jib, and distinguish between ties and struts.

(I.C.E., Oct., 1903.)

2. Three poles, each of length 6 feet, form a tripod, the feet of the poles being 5, 6, and 7 feet apart. Estimate the forces in each leg when a weight of 5 tons is suspended from the apex.

(I.C.E., Feb., 1904.)

3. The tie and jib of a crane are inclined at 60° and 45° respectively to the vertical post, and the direction of the chain is such that it bisects the

angle between them. If the tension in the chain is, by means of tackle, made half the weight carried, find the forces in the tie and jib when the load carried is 10 tons.

(I.C.E., Feb., 1904.)

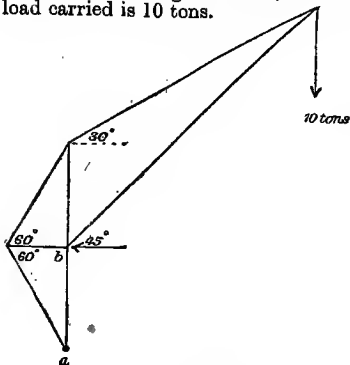


Fig. for Question 1.

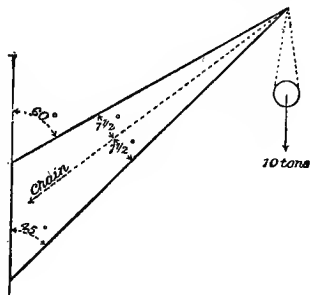


Fig. for Question 3.

4. In the figure, A B is a vertical post rigidly fixed to the ground at B. The jib C E is hinged to A B at C, and is tied to the top of the post by the tie A D, a weight W being supported from E. Indicate by sketches the direction of the reaction at C, and the curves of shearing force and bending moment for the jib and post.

(I.C.E., Oct., 1905.)

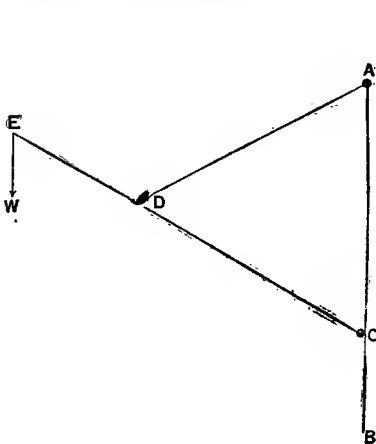


Fig. for Question 4.

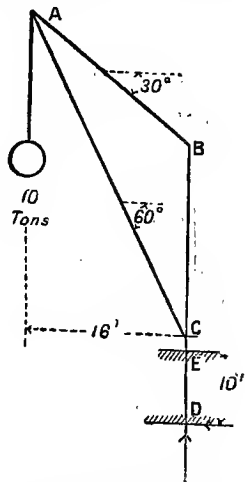


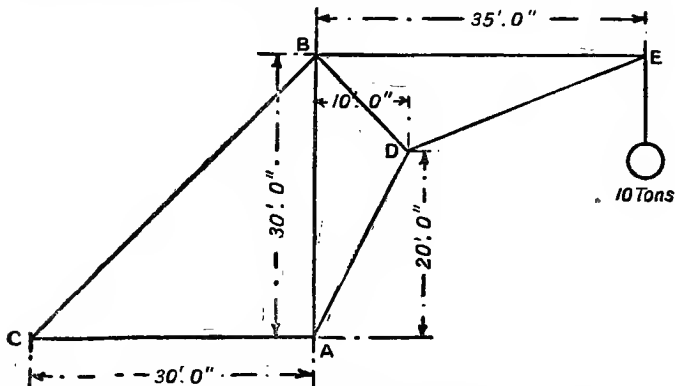
Fig. for Question 5.

5. A jib and pillar crane, as in diagram, carries a load of 10 tons suspended from A at a distance of 16 feet from the axis of post BD.

Neglecting the weight of crane, find the stress in A C and A B. Determine the greatest and least intensities of stress on the cross-section at E, where the post is a hollow casting of 24 inches outside diameter, and 2 inches thick. What are the horizontal and vertical reactions at D?

(I.C.E., Oct., 1907.)

6. A crane is constructed of the form shown by the diagram; the chain is removed, and a test load of 10 tons is suspended from the point E.



Neglecting the weight of the structure, ascertain the stresses in the various members, and the weight of the counter balance weight at C necessary to give an overturning factor of safety of 2.

(I.C.E., Oct., 1907.)

7. The legs of a pair of shear legs are 60 feet long and spaced 20 feet apart at the base; the backstay is 80 feet long. A load of 20 tons is suspended 15 feet in front of the hinges at the base of the legs; find the stresses in the legs and in the backstay.

(I.C.E., Feb., 1908.)

8. A large crane revolves on the top of a braced tower 140 feet high. The tower consists of four steel columns forming a square of 40 feet sides, between centres of columns. Total weight of crane superstructure and tower is 900 tons, and their centre of gravity is 5 feet from the centre of the base of the tower. The wind-pressure on the whole structure amounts to 260 tons, and its resultant acts at 130 feet above the base of the tower. Calculate the downward pressure on the leeward column feet, and the tension, if any, on the windward column feet, for the two conditions:—
(a) Wind acting at right angles to side of tower, and centre of gravity to leeward. (b) Wind acting along diagonal of tower, and centre of gravity to leeward.

(I.C.E., Oct., 1908.)

LECTURE V.—I.C.E. QUESTIONS.

1. A small double-flanged cast-iron girder is of the section shown. Find the distance of the centre of gravity of the section from the bottom of the lower flange.

(I.C.E., Oct., 1903.)

2. A channel section has a base of 10 inches, sides 3 inches, and thickness of metal $\frac{1}{8}$ inch. Find the moment of inertia of the section round a line through the centre of gravity parallel to the base. (I.C.E., Feb., 1904.)

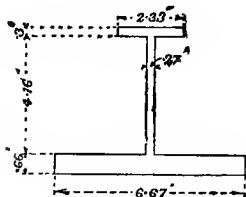


Fig. for Question 1.

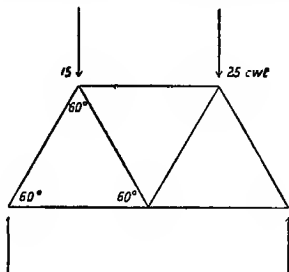


Fig. for Question 4.

3. A girder of 120 feet span consists of 10 equal panels of 12 feet each, with a uniform depth of 12 feet, and with a web bracing of vertical posts and inclined ties. A total load of 30 tons is imposed at the top of each vertical. Take the third panel from either end, and find the stress in each of the four sides of the panel, and also in the diagonal tie.

(I.C.E., Oct., 1904.)

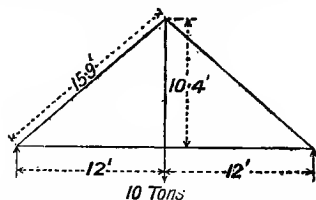
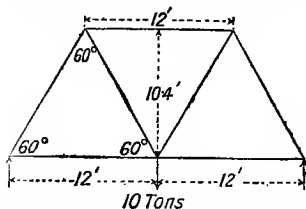
4. A simple Warren girder is as shown in the figure. Estimate, graphically or otherwise, the forces in the different members, and distinguish between ties and struts.

(I.C.E., Feb., 1905.)

5. State the rule for obtaining the position of the centre of gravity of any number of areas situated in one plane, and use it to determine the centre of gravity of the section of a cast-iron beam in which the upper flange is 3 inches wide and 2 inches deep, the web is 3 inches deep and 1 inch thick, and the lower flange is 5 inches wide and 2.5 inches deep.

(I.C.E., Oct., 1905.)

6. Define "force." What elements must be specified to distinguish a given force from all others? Find, otherwise than graphically, the



stresses in the various members of the two trusses sketched. Hence compare the total weights of the trusses, and explain the disparity in these weights.

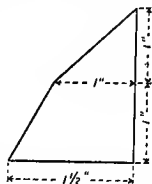
(I.C.E., Oct., 1906.)

7. Sketch the ordinary cross-section given to the flanges in an N-type girder, and indicate how the vertical and diagonal web members are attached to them. Why does the use of a connecting-plate reduce the

number of rivet holes necessary to be drilled through the flange to make the connection? (I.C.E., Oct., 1906.)

8. If an N girder, 120 feet long and 12 feet deep, with 10 bays, has to support a moving uniform load of 2 tons per foot run, find approximately the maximum stresses in the verticals due to this load. (I.C.E., Oct., 1906.)

9. A braced cantilever 30 feet long on the bottom boom $\times$ 10 feet deep, of the N-truss type, having two vertical members and three diagonals, has a load suspended from the extreme end. Assuming the tensile stress to be 6 tons per square inch, and the compression stress 5 tons per square inch, what is the deflection at the point from which the load is suspended? (I.C.E., Feb., 1907.)

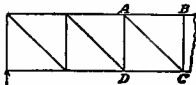


10. Find the position of the centre of gravity of the area shown in the figure. (I.C.E., Feb., 1908.)

LECTURE VIII.—I.C.E. QUESTIONS.

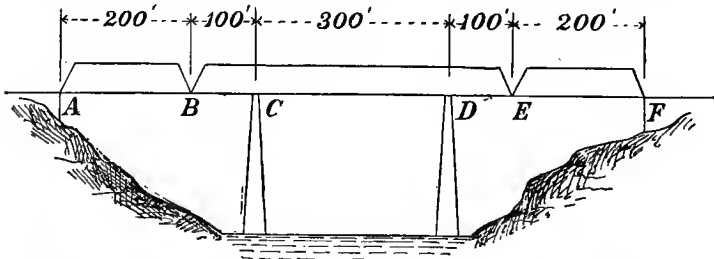
1. A suspension footbridge carried on two chains hanging in a parabolic curve is subject to a load of 1,200 lbs. per foot run. The distance between the piers is 80 feet, the sag is 10 feet, and the chains reach and leave the piers at equal angles. Find the maximum tension in each chain and the pressure on the piers. (I.C.E., Oct., 1903.)

2. An N girder of 90 feet span carries a distributed load of $1\frac{1}{2}$ tons per foot run on the bottom boom. There are six panels and the diagonals are inclined at angles of 45° . Find, by the method of sections or otherwise, the stresses in the sides and diagonal of the panel A B C D. (I.C.E., Feb., 1904.)



3. The uniform section of a rolled iron beam A B C, with two equal flanges, has a depth of 12 inches and a moment of inertia I (about neutral axis) of 240 inch-units. The beam is anchored upon a pier at A, and supported at B, the lengths A B and B C being each 20 feet; and carries a uniform load of 2 cwts. per foot covering its whole length A C. Calculate the deflection of the unsupported end C below the horizontal line A B produced: assuming $E = 12,000$. (I.C.E., Oct., 1904.)

4. In the case of the cantilever beam described in the above question, find the slope of the beam at A, at B, and at C, giving its inclination to the horizontal line A B produced. (I.C.E., Oct., 1904.)



5. A cantilever bridge A B C D E F crosses the three equal spans A C, C D, and D F of 300 feet each. The girder is virtually hinged at B and E,

so that A B and E F are detached girders of 500 feet each, resting upon the cantilever arms C B and D E of 100 feet each, while the girder B C D E is continuous for its whole length of 500 feet. Assume a dead load of $1\frac{1}{2}$ tons per foot throughout, and find the moments at C and D, and at the middle of the central span. (I.C.E., Oct., 1904.)

6. In the bridge above described, assume a live load of 2 tons per foot in addition to the dead load, and find again the moments at C and D and at the middle of the central span under the following conditions:—(a) Live load covering the span C D and no other; (b) live load covering spans A C and D F and no other. (I.C.E., Oct., 1904.)

7. The chain of a suspension bridge crosses a span of 600 feet with a dip of 50 feet, lying in a parabolic curve. Find the tensile stress at the centre and at each end of the span for a maximum load of $1\frac{1}{2}$ tons per foot uniformly distributed, and give a dimensioned sketch design for the steel links and pin-connections. (I.C.E., Oct., 1904.)

8. The chains of the above-named bridge are carried over the towers upon roller bearings, and are continued as blackstays lying in nearly straight lines at an inclination of 3 to 1. For the load given in the previous question find the total weight carried upon the top of each tower, and also the pull in the backstay. (I.C.E., Oct., 1904.)

9. Calculate the two pier moments and the four reactions in a continuous girder bridge having three equal spans of 60 feet each, with a uniform section, when covered by a uniform load of 2 tons per lineal foot, and show how the results are obtained. (I.C.E., Oct., 1904.)

10. Find the pier moments and reactions in the same example, when the uniform load of 2 tons per foot covers the central span, with only 1 ton per foot upon the side spans. (I.C.E., Oct., 1904.)

11. A parabolic arched rib of steel is hinged at each end, but not at the crown. The centre line coincides with its neutral axis. Supposing the abutments to be immovable, what kind of bending stress will be induced by a change of temperature? and how will it be distributed? (I.C.E., Feb., 1905.)

12. The steel cross-girder in a single line bridge spans a width of 15 feet, and carries, by a pair of stringers 5 feet apart, a rolling load of 60 tons and a dead load of 10 tons. Design a suitable cross-section, and show how the shearing forces are to be borne by the riveting. (I.C.E., Feb., 1905.)

13. A block of masonry in the form of a cube of 4 feet edge rests on a wall 3 feet wide, being flush with the wall at one side, and overhanging 1 foot at the other. If the masonry weighs 120 lbs. per cubic foot, what are the greatest and least intensities of pressure on the wall? (I.C.E., Oct., 1905.)

14. A suspension bridge, of 500 feet span between the centres of the towers, is built with a single cable on each side of the roadway, hanging vertically; the angle of the backstays is 45° ; the versed sine of the cable is 50 feet; the distributed load on each cable is 1,000 tons. What is the tension of the cable at the centre of the span, next the towers, and in the backstays? (I.C.E., Oct., 1905.)

15. Sketch two bowstring girders, showing the web-bracing you would adopt when one has to carry a steady, uniformly-distributed load, and the other a travelling load in addition to the dead load. Without calculating the actual stresses, explain the duty each member has to perform, and whether it is in tension or compression. (I.C.E., Oct., 1905.)

16. A plate girder with single web, 50 feet in clear span and 5 feet deep, has to carry a distributed load of 200 tons. Sketch one-half the side elevation, showing the arrangement and dimensions of the web plate and

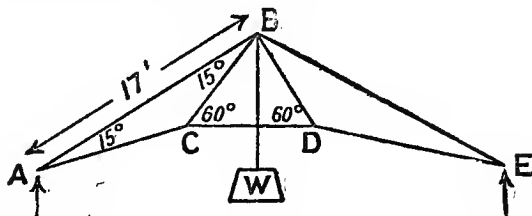
stiffeners, showing how the thickness of the web plate is obtained, and the object of the stiffeners. The top and bottom flanges need not be calculated. (I.C.E., Oct., 1905.)

17. The flooring of a roadway bridge is laid upon square, flat plates $\frac{3}{8}$ inch thick, securely riveted on all sides to strong framing 3 feet apart each way, the unsupported surface being thus 3 feet by 3 feet. What load will the plate carry at the centre, so that the unit stress shall not exceed 6 tons per square inch? (I.C.E., Oct., 1905.)

18. Does the weight of the backstay of a suspension bridge increase or relieve the pull on the anchorage? Show how you arrive at your conclusion. (I.C.E., Feb., 1906.)

19. Discuss the effect of a load on a bridge when suddenly applied, such as a locomotive crossing at a speed of 60 miles an hour. How would the effect of this load compare with that of the engine when stationary, say in the centre of the bridge, and what allowance, if any, would you make in calculating the sectional areas of the various members? (I.C.E., Oct., 1906.)

20. State and prove the theorem known as the Triangle of Forces. Calculate, otherwise than graphically, the stresses in the members AB,



BC, AC, and CD of this truss, when a load of 500 lbs. is slung from the apex B, and state which are struts and which ties. (I.C.E., Feb., 1907.)

21. An N or Pratt type of parallel girder bridge of seven bays is 140 feet span and 15 feet deep. The weight of the main girders works out to be 0.7 ton per lineal foot, and the weight of the platform, which is carried on the upper flange, is 0.55 ton per lineal foot; find the stresses due to this dead load in each member of the bridge, taking half the weight of the main girders at the top and half at the bottom joints. (I.C.E., Feb., 1907.)

22. If a live load of 3 tons per foot run passes over the bridge in the last question, find the maximum stresses in the inclined braces in the centre bay and in the verticals attached to them, due to it. (I.C.E., Feb., 1907.)

23. A suspension bridge has a span of 500 feet between the centres of the towers; the cables have a dip of 50 feet at the centre; the backstays—assumed to be straight—have an inclination of 45° . Neglecting the weight of the structure, what are the stresses in the cable at the centre, at the towers, and in the backstays, when a load of 30 tons is concentrated on each cable (a) at the centre of the span, (b) at one-quarter the span. (I.C.E., Oct., 1907.)

24. Assuming the cables in the previous question to be single, and continuous between the anchorages, sketch the arrangement to be adopted at the towers to allow for extension and contraction of the cables due to temperature and load, and the fact of the backstay having a different angle from that of the cable between the towers. (I.C.E., Oct., 1907.)

25. A suspension bridge of 900 feet span, with a parabolic dip of the cable of 50 feet, is stiffened with a girder 25 feet deep. Assuming the coefficient of expansion to be 0.000007 per 1° F. and the modulus of elasticity as 15,000 tons per square inch, what would be the change of dip with a rise of 50° F., and describe the effect on the flange stress? The girder is not hinged at the centre. (I.C.E., *Feb.*, 1908.)

26. The main girders of a bridge of 100 feet effective span are of common N type with vertical posts and inclined ties at an angle of 45° . There are ten panels in each girder. The two centre panels only are counter-braced. All the diagonal ties are unstiffened flat bars or thin plates. The bridge is designed to have an ample factor of safety against the normal permanent load of 7 tons per panel-point, together with a moving load of 12 tons per panel-point. What influence would an increase of 50 per cent. in the moving load alone have upon the safety of the diagonal ties most affected? Give calculations to support your conclusions. (I.C.E., *Oct.*, 1908.)

APPENDIX B.—(Continued.)

ANSWERS TO QUESTIONS AT END OF LECTURES.

LECTURE I.—ORDINARY.

9. 50 lbs. making an angle 31° to A D.
 10. 41 lbs. through the centre of parallelogram; $2\frac{1}{2}$ lbs. along the bar D C.

LECTURE I.—INST.C.E.

1. Supporting Forces are $17\frac{1}{2}$ and $19\frac{1}{2}$ lbs.
 2. 9.2 lbs. acting at an angle of 132° with the axis of x.
 6. 6 inches from the sheltered side.

LECTURE II.—ORDINARY.

10. and 11.—

Bar.	Tension.	Compression.	Bar.	Tension.	Compression.
J A	...	10.6 tons.	C N	...	9.5 tons.
J K	...	0.85 "	N M	1.3 tons.	...
J I	9.1 tons.	...	M P	4.4 "	...
B K	...	10.0 "	N O	...	0.85 "
K L	1.3 "	...	D O	...	8.9 "
L M	...	1.7 "	P O	5.75 "	...
L I	7.75 "	...	P I	3.9 "	...

LECTURE III.—INST.C.E.

8. Pull in $AB = 2,050$ lbs.; pull in $BC = 2,400$ lbs.

LECTURE IV.—ORDINARY.

2. (i.) Jib, 21 tons; tie, 9.3 tons; post (pull), 4.25 tons; and post (thrust), 8.5 tons. (ii.) Jib, 14 tons; tie, 2.4 tons; and post (thrust), 8.5 tons.

4. Jib, 83.25 cwt.; tie, 41.5 cwt.; post (thrust), 56.25 cwt.; post (pull), 36.5 cwt.; pressure on end of handle = 70.3 lbs.

5. Jib, 33 tons; tie, 27 tons; back stay, 33 tons; counterbalance, 24 tons.

6. 35 tons and $49\frac{1}{2}$ tons.

8. Case 1—compression in jib = $33\frac{1}{2}$ tons, tension in tie-rod = $27\frac{1}{2}$ tons.

Case 2—compression in jib = $36\frac{1}{2}$ tons, tension in tie-rod = 23 tons.

LECTURE IV.—INST.C.E.

1. Back stay = 1 unit, each leg = .9 unit.

2. Stress in $AC = 12\frac{1}{2}$ tons; stress in $AB = 16$ tons.

3. Stress in $AC = 8.9$ tons, and stress in $BC = 16$ tons.

6. $BW = 7.5$ cwt.

7. (a) Tension in tie, zero; thrust in jib, 7.1 tons. (b) (i.) Thrust in upper bar, 1.5 tons; and in jib, 3.5 tons. (ii.) Thrust in upper bar, 3.5 tons; and jib, 8.6 tons.

8. Stress in each leg = 4 tons.

9. Pull at top bearing = $\frac{3}{4}$ ton per ton of load; resultant force on bottom bearing = $1\frac{1}{2}$ tons by measurement; stress in jib = $2\frac{1}{2}$ tons, and stress in tie = $\frac{3}{4}$ ton.

LECTURE V.—ORDINARY.

11. Reaction at $A = 4.6$ tons, and reaction at $E = 5.4$ tons.

14. Maximum BM , 260 foot-tons, and is directly under the trailing axle of the locomotive. Maximum shearing force occurs at the left-hand pier, and is equal to 28 tons.

LECTURE V.—INST.C.E.

1. Thrust in top boom of second bay from the left end = $35\frac{1}{2}$ tons. Stress in lower boom of second bay from the left end = 37 tons.

2. Maximum $BM = 1,757\frac{1}{3}$ ton-feet.

10. The centre of gravity is moved through $\frac{1}{10}$ foot.

11.—

Reactions.	Bars in Compression.	Bars in Tension.
At C = 36 lbs. ,, D = 64 ,,	Stress in A D = 124 lbs. ,, A B = 75 ,, ,, B C = 45 ,,	Stress in D B = 60 lbs. ,, D C = 27 ,,

13. 4.6 inches from lower side of bottom flange.

15. 6.35 inches from bottom flange.

17.—

Bar.	Stress in lbs.	Bar.	Stress in lbs.
B F	(+) 12,750	D K	(+) 11,750
F A	(-) 9,000	K H	(-) 2,200
C G	(+) 11,750	E L	(+) 12,750
G F	(-) 3,600	L A	(-) 9,000
G H	(-) 2,200	L K	(-) 3,600
H A	(-) 9,750		

21. Pressure at each end of girder = 37.5 tons. Pressure on middle support of girder = 125 tons.

LECTURE VI.—ORDINARY.

4. At wall, B M = - 2,000 lb.-feet; S F = - 400 lbs. At centre, B M = - 1,000 lb.-feet; S F = - 400 lbs.

5. B M at fixed end = - 168 ton-feet; B M at 9 feet from fixed end = - 36 ton-feet.

6. At wall, B M = 1,800 lb.-feet; S F = 600 lbs. At 2 feet from wall, B M = 800 lb.-feet; S F = 400 lbs. At 4 feet from wall, B M = 200 lb.-feet; S F = 200 lbs.

8. B M₀ = - 150 ton-feet; S F₀ = 10 tons. B M₆ = 90 ton-feet; S F₆ = 10 tons. B M₈ = - 70 ton-feet; S F₈ = 10 tons. B M₁₂ = - 32 ton-feet; S F₁₂ = 8 tons. B M₁₈ = 2 ton-feet; S F₁₈ = 2 tons.

10. B M₅ = 71 $\frac{1}{2}$ ton-feet; S F₅ = - 9.4 tons. B M₁₀ = 118 $\frac{3}{4}$ ton-feet; S F₁₀ = - 3 $\frac{3}{8}$ tons. B M₂₀ = 152 $\frac{1}{2}$ ton-feet; S F₂₀ = + $\frac{1}{2}$ ton. B M₃₀ = 116 $\frac{1}{4}$ ton-feet; S F₃₀ = + 3 $\frac{5}{8}$ tons.

11. B M₅ = 93 $\frac{3}{4}$ ton-feet; S F₅ = 15 tons. B M₁₀ = 150 ton-feet; S F₁₀ = 7 $\frac{1}{2}$ tons. B M₁₅ = 168 $\frac{3}{4}$ ton-feet; S F₁₅ = 0 ton.

13. B M₁₂ = 140.4 ton-feet; B M₃₀ = 189 ton-feet; maximum B M = 208.85 ton-feet, and occurs at the point 23.7 feet from the left abutment.

14. Maximum B M = 2,654.2 cwt.-feet; B M_C = 2,635 cwt.-feet; maximum B M is 23 feet from left abutment.

LECTURE VII.—ORDINARY.

1. $BM_C = +5\frac{1}{2}$ ton-feet; $BM_B = -40$ ton-feet; BM at point midway between A and $B = +4$ ton-feet; BM at 2 feet from *fixed* end $= +5.2$ ton-feet and $SF = -.66$ ton; BM at 2 feet from *free* end $= -10$ ton-feet and $SF = -5$ tons.

2. BM at $B = -80$ ton-feet; BM at centre $C = -4$ ton-feet; BM at 5 feet from $A = +1\frac{1}{2}$ ton-feet.

4. BM at centre $= -5\frac{1}{2}$ lb.-feet; SF at centre $= -2\frac{5}{8}$ lbs.

5. $BM_A = -40$ ton-feet; $BM_B = -72$ ton-feet; $BM_C = -6$ ton-feet

6. $BM_A = -40$ ton-feet; $BM_B = -72$ ton-feet; $BM_C = +42$ ton-feet.

9. BM at each end $= -25$ ton-feet; BM at centre $= +25$ ton-feet; BM , 2 feet, $= -5$ ton-feet; BM , 6 feet, $= +5$ ton-feet.

10. Points of contrary flexure are 6.33 feet from each end; $BM_A = BM_B = -15$ ton-feet; $BM_C = +7.5$ ton-feet; BM_{at} $= +5$ ton-feet.

ANSWERS TO QUESTIONS IN APPENDIX B.

LECTURE VIII.—INST. C. E.

1. Max. tension in each chain $= 23.9$ tons; pressure on pier $= 42.85$ tons.

3. Deflection $= 2.4$ inches.

5. BM at centre of middle span $= 5,625$ ton-feet.

14. Horizontal tension of cable at centre of span $= 1,250$ tons; tension next towers $= 1,345$ tons; tension in back stays $= 1,893.3$ tons.

APPENDIX C.

THE INSTITUTION OF CIVIL ENGINEERS' EXAMINATION, FEBRUARY, 1909.

ELECTION OF ASSOCIATE MEMBERS.

THEORY OF STRUCTURES.

Not more than EIGHT questions to be attempted by any Candidate.

1. Describe the various modes of failure to be guarded against in designing a simple plate-girder railway bridge with platform between the lower flanges of the main girders.

2. What are the conditions necessary for complete fixity of ends in a girder spanning a single opening? Are these conditions ever realised in practice? Indicate a method by which such fixity could be ensured.

3. In the case of a cylindrical column, describe the influence of the following factors upon its strength:—
(a) Length in relation to diameter, (b) end conditions as affecting free length, (c) compressive strength of material used in the column (compare cast iron with mild steel), (d) elastic stiffness of the material (compare cast iron with mild steel).

4. What is a "factor of safety"? Explain fully what it covers in the case of (a) a steel lattice girder, (b) a masonry arch, (c) a steam boiler, (d) a wire rope.

5. The centre line of a wall is 2 feet from a given boundary. The load on the footing is 10 tons per lineal foot of wall. The footing first proposed was 4 feet wide, as shown in Fig. 1. To reduce the unit pressure on the foundation it was proposed to widen the footing to $5\frac{1}{2}$ feet, as indicated by

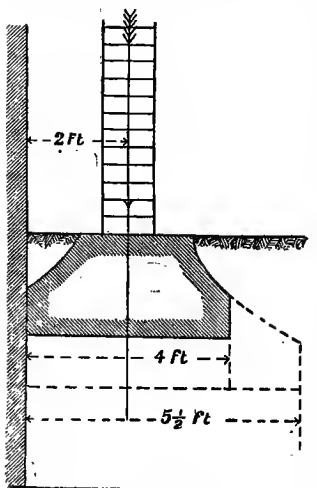


Fig. 1.

dotted lines. Determine the distribution and intensities of pressure for the two cases.

6. In Fig. 2 is shown a portion (not half) of a framed girder. Calculate

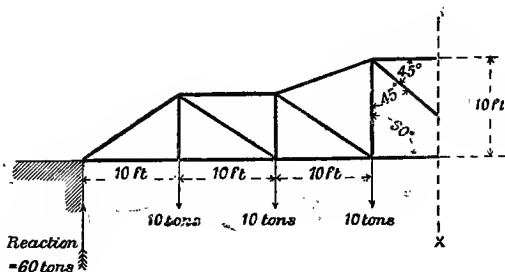


Fig. 2.

the stresses in the top boom and bottom boom and web diagonal cut by the dotted line X X.

7. The five-panel girder shown in Fig. 3 carries a load, W , at 36 feet

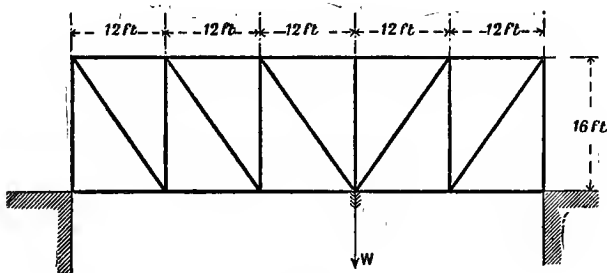


Fig. 3.

from left abutment. The stress in every member is 5 tons per square inch. Calculate the deflection of the girder at the point of application of load. What amount of that deflection is due to web members? Assume the modulus of elasticity to be 10,000 tons per square inch.

8. A continuous gantry girder of uniform section consists of three spans of 48 feet each. The extreme ends are anchored down to prevent uplift. A load of 10 tons is imposed at the middle of the central span. Determine the bending moments at the middle of the central span and over each support. Neglect the weight of girder itself.

9. Describe with sketches the nature of the secondary stresses induced in the members of a common N girder when subjected to load.

10. The cables of a suspension bridge have a span of 300 feet, and a dip of 30 feet. The back stays behind each tower are inclined at 45° to the vertical. The load on the suspended span of 300 feet is 2 tons per lineal foot, inclusive of structure. What is the vertical load on each tower?

11. The rectangular frame (Fig. 4), A B C D, projects from a rigid wall, and carries a load of W tons at C. It is strengthened by adding the dotted diagonal, A C. Each member (inclusive of the new diagonal) has a

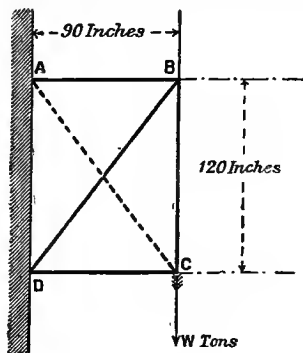


Fig. 4.

section of 10 square inches. The modulus of elasticity is 10,000 tons per square inch. Calculate the load on the bar B C in terms of W after the strengthening.

12. A circular brick chimney, weighing 120 tons, is 100 feet high and 10 feet uniform external diameter. The internal diameter at the base is 6 feet. Assuming a basis wind-pressure of 28 lbs. per square foot on a flat surface, calculate the maximum and minimum intensities of pressure on the brickwork in tons per square foot at the base section.

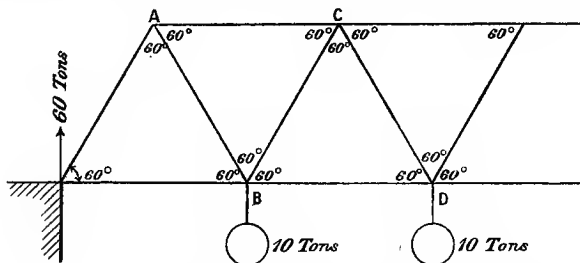
THE INSTITUTION OF CIVIL ENGINEERS' EXAMINATION, OCTOBER, 1909.

ELECTION OF ASSOCIATE MEMBERS.

THEORY OF STRUCTURES.

Not more than EIGHT questions to be attempted by any Candidate

1. Describe the "Method of Sections," and by this principle ascertain the stresses in the bars A B, A C, B C of the frame shown in the figure.



2. A beam 25 feet long is supported at one end and at a point 5 feet from the other end; it is loaded with a distributed load of 10 cwts. per foot run, and has a load of 5 tons suspended from the end of the overhanging portion of the beam. What is the pressure on the supports, and the amount and position of the maximum bending moment? Draw a diagram of the bending moments and shearing forces.

3. A strut 30 feet long is constructed of two channel bars 15 inches $\times$ 4 inches, braced together with diagonal angle bars 3 inches $\times$ 3 inches $\times$ $\frac{1}{2}$ inch (see Fig. on next page). Find the distance marked x , so that the ratio $\frac{l}{r}$ of this length unsupported between the bracing bars does not exceed 60 per cent. of the $\frac{l}{r}$ of the whole column. The maximum moment of inertia of the channel bar is 377 inch-units and the minimum is 14. The centre of gravity is 1 inch from back of channel. Area of channel = 12 square inches, l = length of column, r = radius of gyration.

4. Sketch and dimension two types of movable bearings for the ends of straight girder bridges, assuming the load on the bearing to be 50 tons.

5. Sketch and dimension the joint of boom and bracing bars of a braced girder bridge, as shown in the figure. The top boom consists of two channel bars 15 inches $\times$ 4 inches $\times$ $\frac{5}{8}$ inch, placed 9 inches apart back to back; the verticals consist of two channel bars 10 inches $\times$ 4 inches $\times$ $\frac{5}{8}$ inch, and the diagonal tension bars consist of two flat bars 7 inches $\times$ $\frac{1}{2}$ inch.

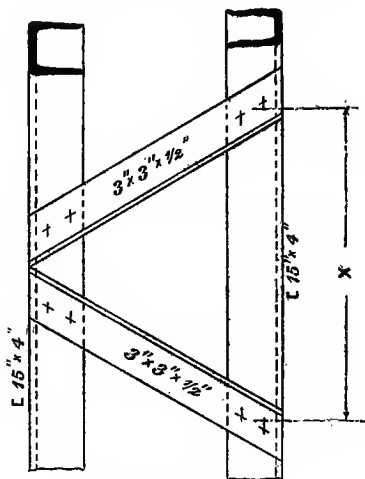


Fig. for Question 3.

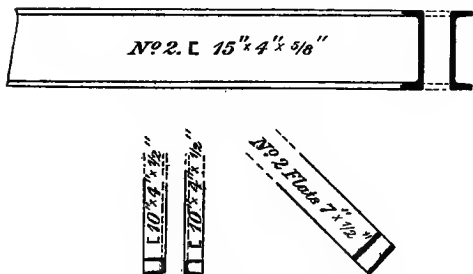


Fig. for Question 5.

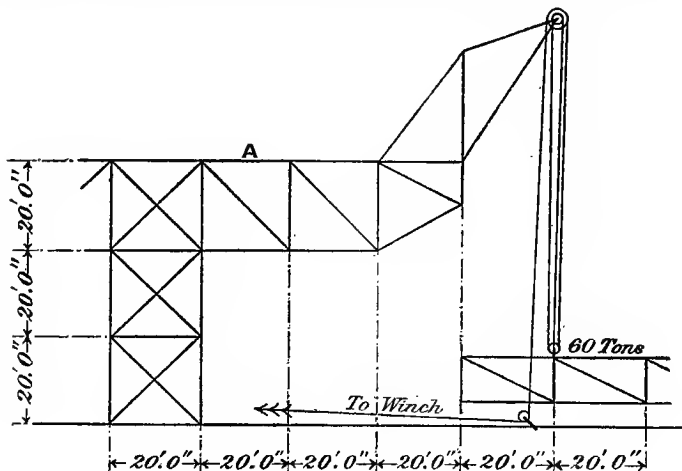
6. A strut or column 10 feet long consists of two T bars 5 inches $\times$ $2\frac{1}{2}$ inches $\times$ $\frac{1}{2}$ inch riveted together back to back; adopting the formula $P = 6 - \frac{1}{40} \frac{l}{r}$ as the allowable stress per square inch on the column, what load will the column stand so as not to exceed this stress? Assume the

corners of the T bars to be rectangular and the sides parallel. What do you consider the maximum $\frac{l}{r}$ ratio allowable? l = length of column, r = radius of gyration.

7. Compare the respective advantages and disadvantages of the two types of bascule bridges known as the "Trunnion" and the "Scherzer rolling lift bascule."

8. Describe two methods of stiffening the deck of a suspension bridge from vertical oscillations, and state the advantages and disadvantages of each.

9. The central portion of a cantilever bridge is being lifted into position,



as shown in the figure. Neglecting inertia and friction, what is the stress in the member marked A, the suspended load on the tackle being 60 tons?

10. The dock gates of a 60-foot entrance are segmental in plan, having a versed sine of 6 feet. With a depth of water of 20 feet, what is the pressure on a strip 12 inches wide vertically on the hollow quoins and at the jointing of the gates, the centre of the strip to be at a distance of 5 feet from the bottom? Assume weight of water to be 64 lbs. per cubic foot.

11. Give a *résumé* of the results of Wöhler's or similar experiments on repeated application of a load, and state how these results affect the designing of girder bridges.

12. A concrete dam is triangular in section and is built of stone, having a density 2.24 compared with water. What must be the thickness of the dam at a depth of 30 feet, if the line of resultant pressure is to cut the section at a distance of two-thirds of its breadth from the vertical inner face, with a full head of water? What is the pressure on the toe of the dam on opposite side of reservoir when the dam is empty, assuming the pressure on the bed to be uniformly varying?

THE INSTITUTION OF CIVIL ENGINEERS'

EXAMINATION, FEBRUARY, 1910.

ELECTION OF ASSOCIATE MEMBERS.

THEORY OF STRUCTURES.

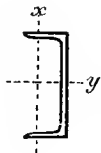
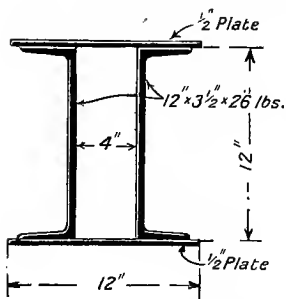
Not more than EIGHT questions to be attempted by any Candidate.

1. A continuous girder, of length L , rests upon four supports on the same level, the spans being equal. A load, W , is placed on the centre of each span; find the pressure on each of the supports, and draw the diagrams of the bending moments and shearing stresses. A hinge to be placed at a point one-ninth of the span on the abutment side of the centre piers; but no hinge in central span.

2. A suspension bridge has a span of 300 feet between the centre of the towers, the dip of the cable being 30 feet; the backstays are anchored at a point 100 feet from the centre of the tower, and 50 feet below the cable at the top of the towers. The dead load is equal to 1 ton per foot run on each cable. What is the vertical pressure on the piers; and what are the stresses in the cable at the centre of the span, at the top of the piers, and in the backstays?

3. In a suspension bridge as described in Question No. 2, sketch the plans generally adopted for making the action of the load on the top of the piers vertical, stating the necessary conditions to attain this object.

4. A steel column is constructed of the following section, as shown in the figure, consisting of two channel bars, 12 inches by $3\frac{1}{2}$ inches by 26.1 lbs., spaced 4 inches apart, back to back, and plated with 12-inch by $\frac{1}{2}$ -inch plates. Adopting the



formula $P = 6 - \frac{1}{40} \frac{l}{r}$ as

the allowable stress per square inch on the column, find the load on a column 20 feet long which will not exceed this stress. Sectional area of channel bar = 7.6 square inches. Moment of inertia, $I_x = 7.5$, $I_y = 158$.

The distance of the centre of gravity from back of channel is 0.8 inch. l = length of column, r = radius of gyration.

5. Adopting the channel bars described in the above Question No. 4, calculate the distance between the backs of the channel bars so that the

of 15 feet, being well built in at the ends. It supports a wall of brickwork 20 feet high, 27 inches thick. A cubic foot of brickwork weighs 112 lbs. Draw the diagram of bending moment; what is the greatest bending moment? (B. of E., S. 2, Div. *a*, 1910.)

44. Choose any three forces not meeting at a point and not parallel to one another. Show how we find, graphically, their resultant or their equilibrant. (B. of E., S. 2, Div. *a*, 1910.)

45. Consider a high house of residential flats; the total horizontal section *S* of all the walls just below a floor, at a height *h* above the ground, gets less as *h* is greater. Why is this, and state the roughly correct law that connects *S* and *h*? (B. of E., S. 2, Div. *a*, 1910.)

STAGE 3.

You must not attempt more than EIGHT questions in all, including Nos. 51 and 52—that is to say, although 51 and 52 are not compulsory, you are not allowed to take more than six questions in addition to Nos. 51 and 52.

53. Each of the legs of a pair of sheer legs is 45 feet long; they are spread out 23 feet at their base. The length of the back-stay is 60 feet. If a load of 40 tons is being lifted at a distance of 15 feet, measured in a perpendicular line from the line joining the feet of the two legs, find the forces in the legs and in the back-stay due to this load.

(B. of E., S. 3, Div. *a*, 1910.)

54. Concrete exerts on earth at the bottom of a trench a downward pressure of 2 tons per square foot; the earth weighs 130 lbs. per cubic foot, and its angle of repose (in Rankine's theory) is 30° ; what is the least safe depth below the earth's natural surface of the bottom of the concrete?

Why are we unable to make much practical use of the theory of earth pressure? (B. of E., S. 3, Div. *a*, 1910.)

56. How do we find the stress everywhere in the cross-section of a flying buttress or revetment wall or arched rib, given the resultant of all the forces which act on the structure on one side of the section? Prove your method to be correct. (B. of E., S. 3, Div. *a*, 1910.)

57. A uniform beam, 30 feet long, fixed at the ends, has a load of 20 tons spread uniformly along it. It has also two loads of 5 tons each hung from points which are 10 feet from the ends. What is the bending moment everywhere, and what is its greatest value? (B. of E., S. 3, Div. *a*, 1910.)

59. A number of forces in a plane not meeting in a point. Treating them graphically, what is the condition of equilibrium? Prove your statement to be correct. (B. of E., S. 3, Div. *a*, 1910.)

APPENDIX D.

EXAMINATION TABLES.

USEFUL CONSTANTS.

1 Inch = 25·4 millimetres.

1 Gallon = ·1605 cubic foot = 10 lbs. of water at 62° F. ∴ 1 lb. = ·01605 cubic foot.

1 Knot = 6080 feet per hour. 1 Naut = 6080 feet.

Weight of 1 lb. in London = 445,000 dynes.

One pound avoirdupois = 7000 grains = 453·6 grammes.

1 Cubic foot of water weighs 62·3 lbs.

1 Cubic foot of air at 0° C. and 1 atmosphere, weighs ·0807 lb.

1 Cubic foot of Hydrogen at 0° C. and 1 atmosphere, weighs ·00557 lb.

1 Foot-pound = $1·3562 \times 10^7$ ergs.

1 Horse-power-hour = 33000 × 60 foot-pounds.

1 Electrical unit = 1000 watt-hours.

Joule's Equivalent to suit Regnault's H, is $\begin{cases} 774 \text{ ft.-lbs.} = 1 \text{ Fah. unit.} \\ 1393 \text{ ft.-lbs.} = 1 \text{ Cent. } \end{cases}$

1 Horse-power = 33000 foot-pounds per minute = 746 watts.

Volts × amperes = watts.

1 Atmosphere = 14·7 lb. per square inch = 2116 lbs. per square foot = 760 m.m. of mercury = 10^6 dynes per sq. cm. nearly.

A Column of water 2·3 feet high corresponds to a pressure of 1 lb. per square inch.

Absolute temp., $t = \theta^\circ \text{ C.} + 273^\circ\text{·}7$.

Regnault's H = $606^\circ\text{·}5 + \cdot 305 \theta^\circ \text{ C.} = 1082 + \cdot 305 \theta^\circ \text{ F.}$

$p u^{1\cdot 0646} = 479$

$\log_{10} p = 6\cdot 1007 - \frac{B}{t} - \frac{C}{t^2}$

where $\log_{10} B = 3\cdot 1812$, $\log_{10} C = 5\cdot 0871$,

p is in pounds per square inch, t is absolute temperature Centigrade,

u is the volume in cubic feet per pound of steam.

$\pi = 3\cdot 1416 = \frac{22}{7} = \frac{355}{113} = 10(\sqrt{3} - \sqrt{2})$.

One radian = 57·3 degrees.

To convert common into Napierian logarithms, multiply by 2·3026.

The base of the Napierian logarithm is $e = 2\cdot 7183$.

The value of g at London = 32·182 feet per second per second.

TABLE OF LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 13 16	18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5659	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6876	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7136	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7186	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7276	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7

TABLE OF LOGARITHMS.—Continued.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7876	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8076	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9406	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

TABLE OF ANTILOGARITHMS.

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·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

TABLE OF ANTILOGARITHMS.—Continued.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
*60	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	6	6	7
*61	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
*62	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
*63	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
*64	3467	3476	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	6	6	6	7
*65	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	6	6	7	7
*66	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	8	3	4	6	6	7	8
*67	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
*68	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	5	6	7	8	8
*69	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
*70	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	6	6	6	7	8
*71	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
*72	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
*73	4266	4276	4286	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	6	6	7	8	9
*74	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	6	6	7	8	9
*75	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	6	6	7	8	9
*76	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	6	6	7	9	10
*77	4677	4688	4699	4710	4721	4732	4742	4753	4764	4776	1	2	3	4	5	7	8	9	10
*78	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
*79	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
*80	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
*81	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
*82	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
*83	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
*84	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	6	6	8	9	10	12
*85	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	6	7	8	9	10	12
*86	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	6	7	8	9	11	12
*87	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
*88	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
*89	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
*90	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
*91	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
*92	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
*93	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
*94	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	6	6	8	10	11	13	15
*95	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	6	7	8	10	12	13	16
*96	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	6	7	8	10	12	13	15
*97	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
*98	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
*99	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
*90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
*91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
*92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
*93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
*94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
*95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
*96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
*97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
*98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
*99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE OF FUNCTIONS OF ANGLES.

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees.	Radians.								
0°	0	000	0	0	∞	1	1.414	1.5708	90°
1	.0175	.017	.0175	.0175	57.2900	.9998	1.402	1.5533	89
2	.0349	.035	.0349	.0349	28.6363	.9994	1.389	1.5359	88
3	.0524	.052	.0523	.0524	19.0811	.9986	1.377	1.5184	87
4	.0698	.070	.0698	.0699	14.3007	.9976	1.364	1.5010	86
5	.0873	.087	.0872	.0875	11.4301	.9962	1.351	1.4835	85
6	.1047	.105	.1045	.1051	9.5144	.9945	1.338	1.4661	84
7	.1222	.122	.1219	.1228	8.1443	.9925	1.325	1.4486	83
8	.1396	.140	.1392	.1405	7.1154	.9903	1.312	1.4312	82
9	.1571	.157	.1564	.1584	6.3138	.9877	1.299	1.4137	81
10	.1745	.174	.1736	.1768	5.6713	.9848	1.286	1.3963	80
11	.1929	.192	.1908	.1944	5.1446	.9816	1.272	1.3788	79
12	.2094	.209	.2079	.2126	4.7046	.9781	1.259	1.3614	78
13	.2269	.226	.2260	.2309	4.3315	.9744	1.245	1.3439	77
14	.2443	.244	.2419	.2493	4.0108	.9703	1.231	1.3265	76
15	.2618	.261	.2588	.2679	3.7321	.9659	1.218	1.3090	75
16	.2793	.278	.2756	.2867	3.4874	.9613	1.204	1.2915	74
17	.2967	.296	.2924	.3057	3.2709	.9563	1.190	1.2741	73
18	.3142	.313	.3090	.3249	3.0777	.9511	1.176	1.2566	72
19	.3316	.330	.3256	.3443	2.9042	.9455	1.161	1.2392	71
20	.3491	.347	.3420	.3640	2.7475	.9397	1.147	1.2217	70
21	.3665	.364	.3584	.3839	2.6051	.9336	1.133	1.2043	69
22	.3840	.382	.3746	.4040	2.4751	.9272	1.118	1.1868	68
23	.4014	.399	.3907	.4245	2.3559	.9205	1.104	1.1694	67
24	.4189	.416	.4067	.4452	2.2460	.9135	1.089	1.1519	66
25	.4363	.433	.4226	.4663	2.1445	.9063	1.075	1.1345	65
26	.4538	.450	.4384	.4877	2.0503	.8988	1.060	1.1170	64
27	.4712	.467	.4540	.5096	1.9626	.8910	1.046	1.0996	63
28	.4887	.484	.4695	.5317	1.8807	.8829	1.030	1.0821	62
29	.5061	.501	.4848	.5543	1.8040	.8746	1.015	1.0647	61
30	.5236	.518	.5000	.5774	1.7321	.8660	1.000	1.0472	60
31	.5411	.534	.5150	.6009	1.6643	.8572	.985	1.0297	59
32	.5585	.551	.5299	.6249	1.6003	.8480	.970	1.0123	58
33	.5760	.568	.5446	.6494	1.5399	.8387	.954	.9948	57
34	.5934	.586	.5592	.6745	1.4826	.8290	.939	.9774	56
35	.6109	.601	.5736	.7002	1.4281	.8192	.923	.9599	55
36	.6283	.618	.5878	.7265	1.3764	.8090	.908	.9425	54
37	.6458	.635	.6018	.7536	1.3270	.7986	.892	.9250	53
38	.6632	.651	.6157	.7813	1.2799	.7880	.877	.9076	52
39	.6807	.668	.6293	.8098	1.2349	.7771	.861	.8901	51
40	.6981	.684	.6428	.8391	1.1918	.7660	.845	.8727	50
41	.7156	.700	.6561	.8693	1.1504	.7547	.829	.8552	49
42	.7330	.717	.6691	.9004	1.1106	.7431	.813	.8378	48
43	.7505	.733	.6820	.9325	1.0724	.7314	.797	.8203	47
44	.7679	.749	.6947	.9657	1.0355	.7193	.781	.8029	46
45°	.7854	.766	.7071	1.0000	1.0000	.7071	.765	.7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

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